

# Logic

CSCI 4511/6511

Joe Goldfrank

# Announcements

- Homework 2 is due on 29 September at 11:55 PM
  - Bug!
- Midterm Exam - 16 Oct
  - In class
  - Open note

# Review

# CSPs

- State variables:  $X_1, X_2, \dots, X_n$
- State variable domains:  $D_1, D_2, \dots, D_n$ 
  - The domain specifies which values are permitted for the state variable
  - Domain: set of allowable variables (or permissible range for continuous variables)<sup>1</sup>
  - Some constraints  $C_1, C_2, \dots, C_m$  restrict allowable values

1. Or a hybrid, such as a union of ranges of continuous variables.

# CSP Constraints

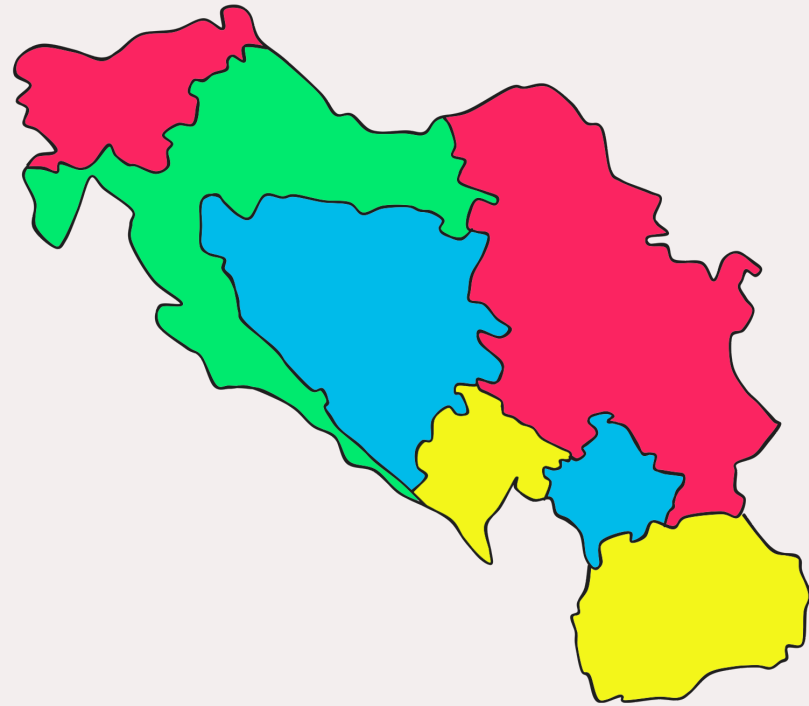
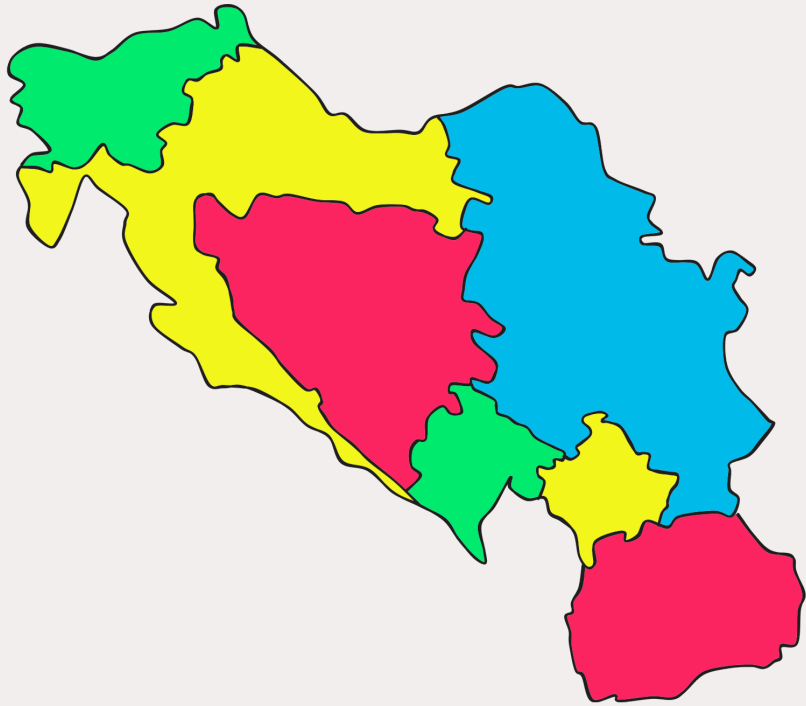
- $X_1$  and  $X_2$  both have real domains, i.e.  $X_1, X_2 \in \mathbb{R}$ 
  - A constraint could be  $X_1 < X_2$
- $X_1$  could have domain {red, green, blue} and  $X_2$  could have domain {green, blue, orange}
  - A constraint could be  $X_1 \neq X_2$
- $X_1, X_2, \dots, X_{100} \in \mathbb{R}$ 
  - Constraint: exactly four of  $X_i$  equal 12
  - Rewrite as binary constraint?

# Assignments

- Assignments must be to values in each variable's domain
- Assignment violates constraints?
  - Consistency
- All variables assigned?
  - Complete

# Four-Colorings

Two possibilities:



# Solving CSPs

- We can search!
  - ...the space of consistent assignments
- Complexity  $O(d^n)$ 
  - Domain size  $d$ , number of nodes  $n$
- Tree search for node assignment
  - Inference to reduce domain size
- Recursive search



# What Even Is Inference

- Constraints on one variable restrict others:
  - $X_1 \in \{A, B, C, D\}$  and  $X_2 \in \{A\}$
  - $X_1 \neq X_2$
  - Inference:  $X_1 \in \{B, C, D\}$
- If an unassigned variable has no domain...
  - Failure

# Ordering

- $\text{SELECT-UNASSIGNED-VARIABLE}(CSP, \textit{assignment})$ 
  - Choose most-constrained variable<sup>1</sup>
- $\text{ORDER-DOMAIN-VARIABLES}(CSP, \textit{var}, \textit{assignment})$ 
  - Least-constraining value
- Why?

1. or MRV: “Minimum Remaining Values”

# Restructuring

Tree-structured CSPs:

- *Linear time* solution
- Directional arc consistency:  $X_i \rightarrow X_{i+1}$
- Cutsets
- Sub-problems

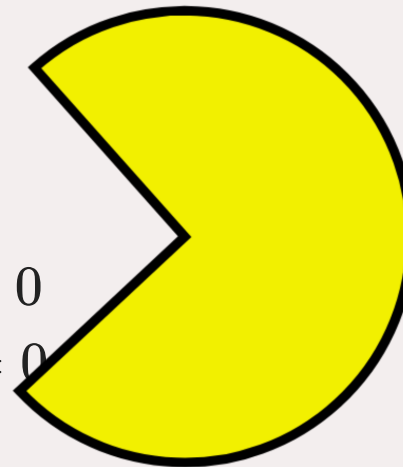
# Continuous Domains

- Linear:

$$\begin{aligned} \max_x \quad & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} \quad & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq 0 \end{aligned}$$

- Convex

$$\begin{aligned} \min_x \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & g_i(\mathbf{x}) \leq 0 \\ & h_i(\mathbf{x}) = 0 \end{aligned}$$



**Logic**

# Yugoslav Logic

$$R_{HK} \Rightarrow \neg R_{SI}$$

$$G_{HK} \Rightarrow \neg G_{SI}$$

$$B_{HK} \Rightarrow \neg B_{SI}$$

$$R_{HK} \vee G_{HK} \vee B_{HK}$$

...

Goal: find assignment of variables that satisfies conditions

# Is It Possible To Know Things?

Yes.



# How Even Do We Know Things?

- What color is an apple?
  - Red?
  - Green?
  - Blue?
- Are you sure?



# Symbols

- Propositional symbols
  - Similar to boolean variables
  - Either True or False

# The Unambiguous Truth

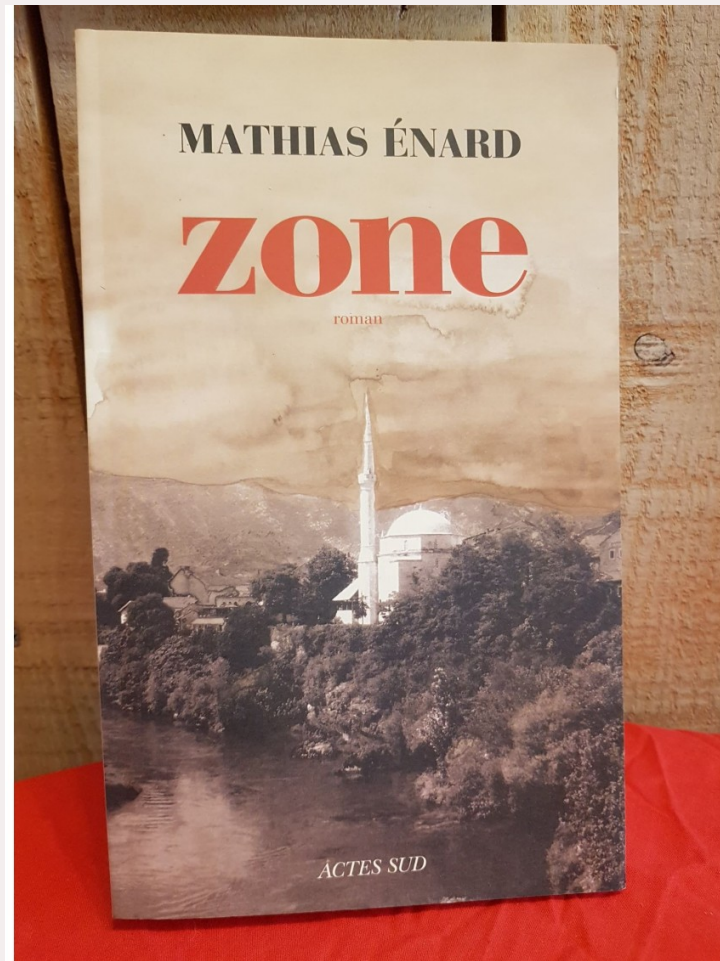
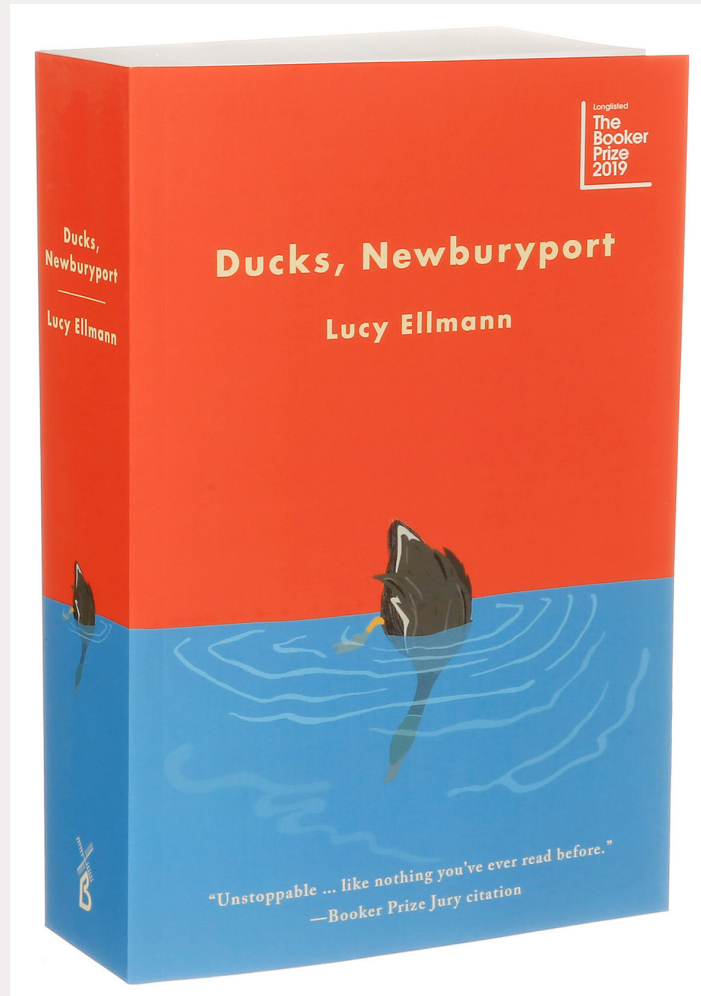
- IT IS A NICE DAY .
  - It is difficult to discern an unambiguous truth value.
- IT IS WARM OUTSIDE .
  - This has some truth value, but it is ambiguous.
- THE TEMPERATURE IS AT LEAST 78°F OUTSIDE .
  - This has an unambiguous truth value.<sup>1</sup>

1. Provided that 'outside' is well-defined.

# What Matters, Matters

- *Non-ambiguity* required
- Arbitrary detail is not
- THE TEMPERATURE IS EXACTLY 78°F OUTSIDE.
  - We don't necessarily need any other “related” symbols
- What is the problem?
- What do we care about?

# Sentences



# Sentences

- What is a linguistic sentence?
  - Subject(s)
  - Verb(s)
  - Object(s)
  - *Relationships*
- What is a logical sentence?
  - Symbols
  - Relationships

# Familiar Logical Operators

- $\neg$ 
  - “Not” operator, same as CS (!, **not**, etc.)
- $\wedge$ 
  - “And” operator, same as CS (&&, **and**, etc.)
  - This is sometimes called a *conjunction*.
- $\vee$ 
  - “Inclusive Or” operator, same as CS.
  - This is sometimes called a *disjunction*.

# Unfamiliar Logical Operators

- $\Rightarrow$

- Logical *implication*.

- If  $X_0 \Rightarrow X_1$ ,  $X_1$  is always True when  $X_0$  is True.

- If  $X_0$  is False, the value of  $X_1$  is not constrained.

- $\iff$

- “If and only If.”

- If  $X_0 \iff X_1$ ,  $X_0$  and  $X_1$  are either both True or both False.

- Also called a *biconditional*.

# Equivalent Statements

- $X_0 \Rightarrow X_1$  alternatively:
  - $(X_0 \wedge X_1) \vee \neg X_0$
- $X_0 \iff X_1$  alternatively:
  - $(X_0 \wedge X_1) \vee (\neg X_0 \wedge \neg X_1)$
- Can we make an XOR?



# Knowledge Base & Queries

- We encode everything that we ‘know’
  - Statements that are true
- We query the knowledge base
  - Statement that we’d like to know about
- Logic:
  - Is statement consistent with KB?

# Models

- Mathematical abstraction of problem
  - Allows us to solve it
- Logic:
  - Set of truth values for all sentences
  - ...sentences comprised of symbols...
  - Set of truth values for all symbols
  - New sentences, symbols over time

# Entailment

- $KB \models A$ 
  - “Knowledge Base entails  $A$ ”
  - For every model in which  $KB$  is True,  $A$  is also True
  - One-way relationship:  $A$  can be True for models where  $KB$  is not True.
- Vocabulary:  $A$  is the *query*

# Knowing Things

Falsehood:

- $KB \models \neg A$ 
  - No model exists where  $KB$  is True and  $A$  is True

It is possible to not know things:<sup>1</sup>

- $KB \not\models A$
- $KB \not\models \neg A$

1.  $\not\models$  – “does not entail”

# It Is Possible To Not Know Things 🙄

I have a plastic platter with eighteen hamburgers on it. I eat one hamburger, rotate the platter upside down, rotate it back rightside up, and offer one hamburger to Alan. How many hamburgers are left on the platter?

Initially, you have 18 hamburgers on the platter. After you eat one, you have:

$18 - 1 = 17$  hamburgers left.

Next, when you rotate the platter upside down and then back to the right side up, the hamburgers stay on the platter. You then offer one hamburger to Alan. So now, you have:

$17 - 1 = 16$  hamburgers left on the platter.

Therefore, there are **16 hamburgers** left on the platter.

# Lexicon

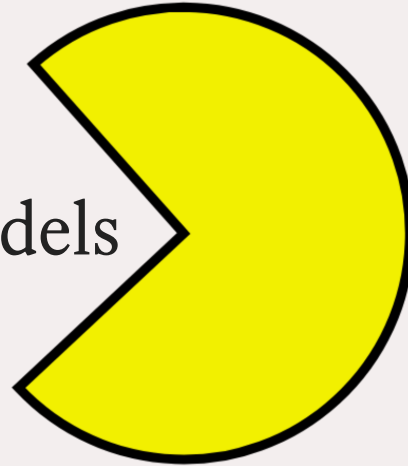
- *Valid*
  - $A \vee \neg A$
- *Satisfiable*
  - True for some models
- *Unsatisfiable*
  - $A \wedge \neg A$

# Inference

- $KB$  models real world
  - Truth values unambiguous
  - $KB$  coded correctly
- $KB \models A$ 
  - $A$  is true in the real world

# Inference - How?

- Model checking
  - Enumerate possible models
  - We can do better
  - NP-complete 😞
- Theorem proving
  - Prove  $KB \models A$





# Satisfiability

- Commonly abbreviated “SAT”
  - Not the Scholastic Assessment Test
  - Much more difficult
  - *First* NP-complete problem
- The

*Deliberate typographical error!*

# Satisfiability

- Commonly abbreviated “SAT”
- $(X_0 \wedge X_1) \vee X_2$ 
  - Satisfied by  $X_0 = \text{True}$ ,  $X_1 = \text{False}$ ,  $X_2 = \text{True}$
  - Satisfied for any  $X_0$  and  $X_1$  if  $X_2 = \text{True}$
- $X_0 \wedge \neg X_0 \wedge X_1$ 
  - Cannot be satisfied by any values of  $X_0$  and  $X_1$

# Satisfaction

- SAT reminiscent of Constraint Satisfaction Problems
- CSPs reduce to SAT
  - Solving SAT  $\rightarrow$  solving CSPs
  - Restricted to specific operators
  - CSP global constraints  $\rightarrow$  refactor as binary
- Still NP-Complete

# Why Do I Keep On Doing This To You

*This is the entire point of the course.*

*Theory and practice are the same, in theory, but in practice they differ.*

# CSP Solution Methods

- They all work
- Backtracking search
- Hill-climbing
- Ordering (?)

# SAT Solvers

- Heuristics
- PicoSAT
  - Python bindings: `pycosat`
  - (Solver written in C) (it's fast)
- You don't have to know anything about the problem
  - This is not actually true
- Conjunctive Normal Form

# Conjunctive Normal Form

- *Literals* — symbols or negated symbols
  - $X_0$  is a literal
  - $\neg X_0$  is a literal
- *Clauses* — combine literals and disjunction using disjunctions ( $\vee$ )
  - $X_0 \vee \neg X_1$  is a valid disjunction
  - $(X_0 \vee \neg X_1) \vee X_2$  is a valid disjunction

# Conjunctive Normal Form

- *Conjunctions* ( $\wedge$ ) combine clauses (and literals)
  - $X_1 \wedge (X_0 \vee \neg X_2)$
- Disjunctions cannot contain conjunctions:
- $X_0 \vee (X_1 \wedge X_2)$  not in CNF
  - Can be rewritten in CNF:  $(X_0 \vee X_1) \wedge (X_0 \vee X_2)$

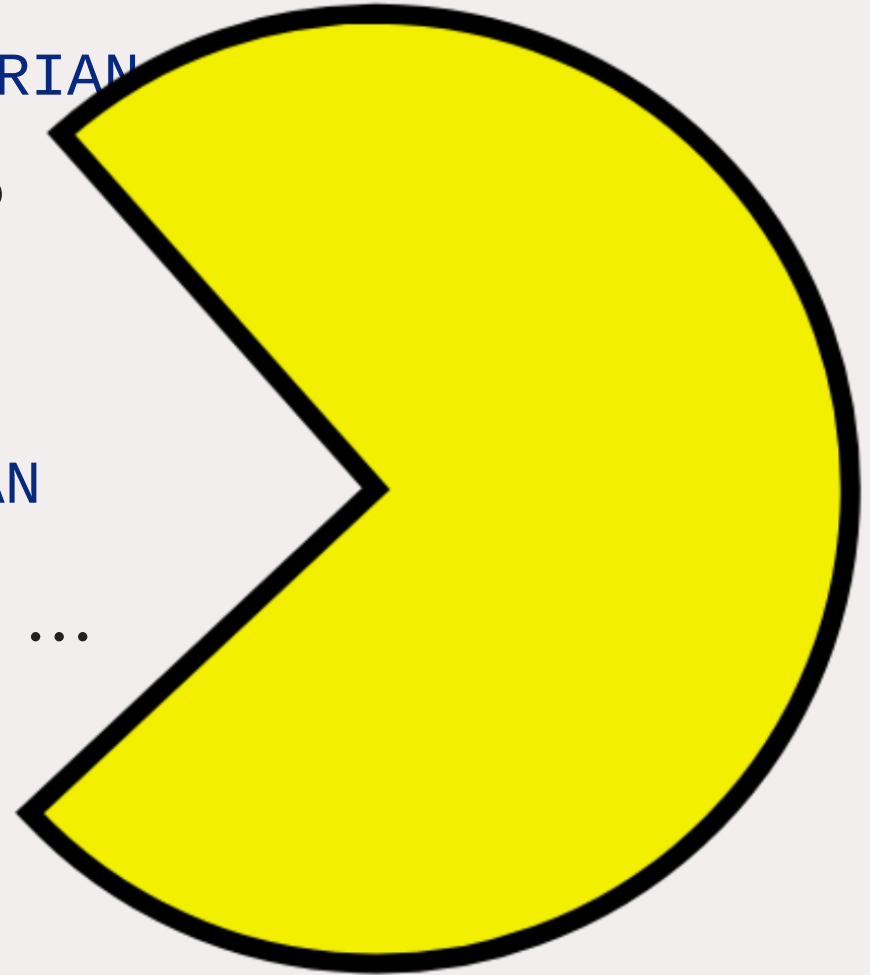


# Converting to CNF

- $X_0 \iff X_1$ 
  - $(X_0 \implies X_1) \wedge (X_1 \implies X_0)$
- $X_0 \implies X_1$ 
  - $\neg X_0 \vee X_1$
- $\neg(X_0 \wedge X_1)$ 
  - $\neg X_0 \vee \neg X_1$
- $\neg(X_0 \vee X_1)$ 
  - $\neg X_0 \wedge \neg X_1$

# Limitations

- Consider: NO CAT IS A VEGETARIAN
- Express in propositional symbols?
- $\neg$  FIRST CAT IS A VEGETARIAN
- $\neg$  SECOND CAT IS A VEGETARIAN
- $\neg$  THIRD CAT IS A VEGETARIAN ...



# Solutions

## First-Order Logic:

- $\forall$  (“for all”)
- $\exists$  (“there exists at least one”)

## Loops 😊 :

```
1 for cat in cats:  
2     t = Expr(f"{cat} is not a vegetarian")  
3     Exprs.push(t)
```

# Probability

# Coin Flip

Three outcomes: HEADS, TAILS, and EDGE.

- If the coin lands heads, it does not land tails or edge:
  - $\text{HEADS} \wedge \neg \text{TAILS} \wedge \neg \text{EDGE}$
- Similarly:
  - $\text{TAILS} \wedge \neg \text{HEADS} \wedge \neg \text{EDGE}$
- &c.
  - $\text{EDGE} \wedge \neg \text{HEADS} \wedge \neg \text{TAILS}$

# Coin Flip

Propositional logic tells us:

$$\begin{aligned} & (\text{HEADS} \wedge \neg \text{TAILS} \wedge \neg \text{EDGE}) \vee (\text{TAILS} \wedge \neg \text{HEADS} \wedge \neg \text{EDGE}) \vee \\ & (\text{EDGE} \wedge \neg \text{HEADS} \wedge \neg \text{TAILS}) \end{aligned}$$

*This is remarkably unsatisfying.*

# Belief

- Heads percentage?
- Tails percentage?
- Edge percentage?

How do you know?

*What does it mean to **know** ?*





# References

- Stuart J. Russell and Peter Norvig. *Artificial Intelligence: A Modern Approach*. 4th Edition, 2020.
- Stanford CS231
- UC Berkeley CS188