Partially-Observable Markov Decision Processes

CSCI 4511/6511

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Announcements

- Homework Four: 11 Nov
- Extra Credit HW: Due 4 Dec (releases next week)
- Project Proposals: 13 Nov
- Final Exam: 4 Dec
- Project Deadline: 13 Dec

MDP Example

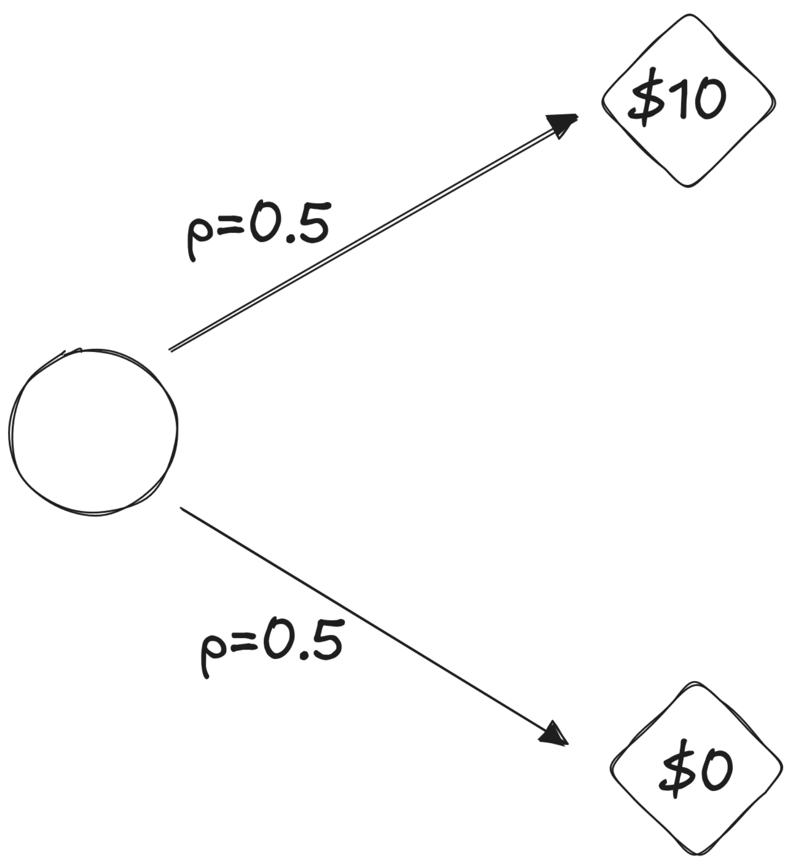
Belief

State Uncertainty

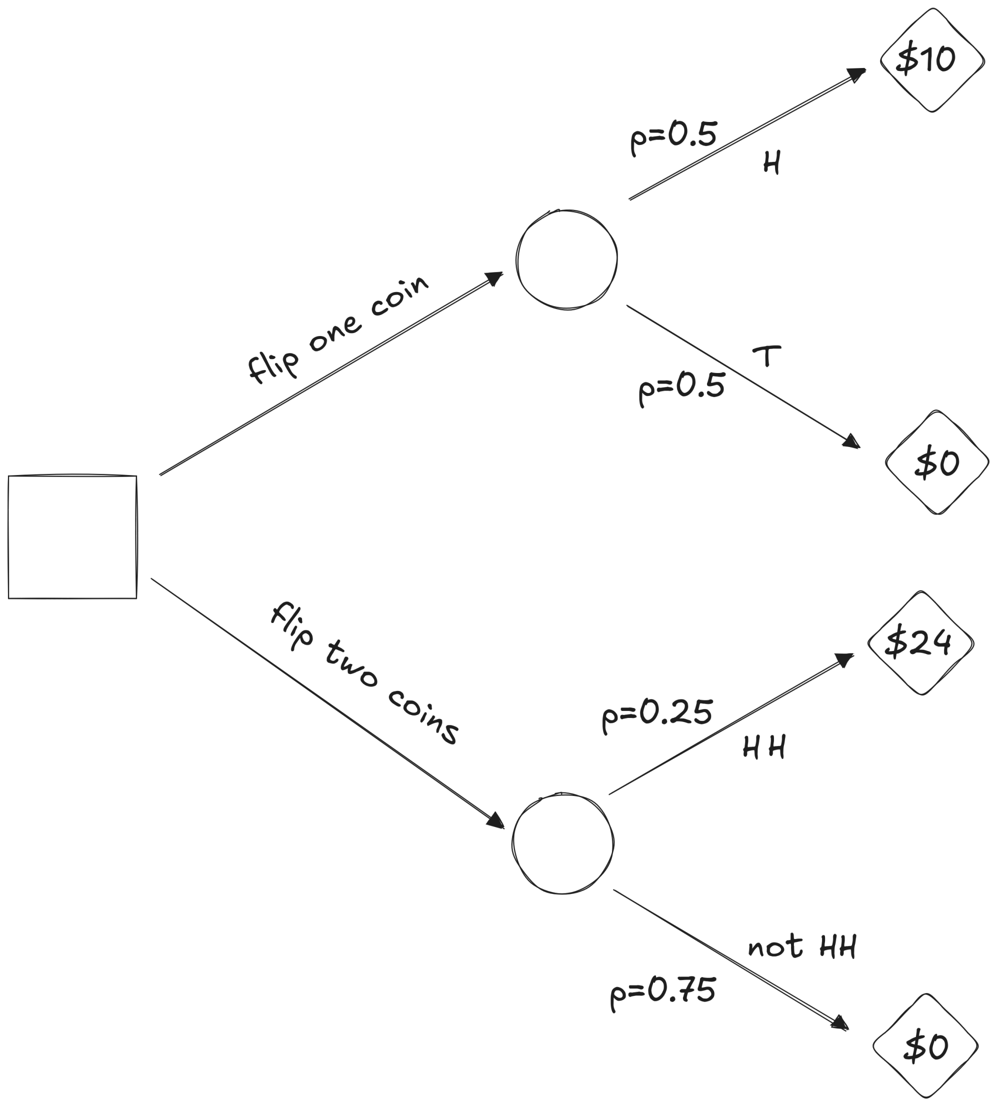
- Markov Decision Process (MDP):
  - Assumes state is observed
  - Policy returns action as function of state
  - Decision-maker observes state, selects action
- Partially-Observable Markov Decision Process (POMDP):
  - State not fully observed
  - Decision-maker observes… *something*
  - That *something* is related to state

Decision Theory

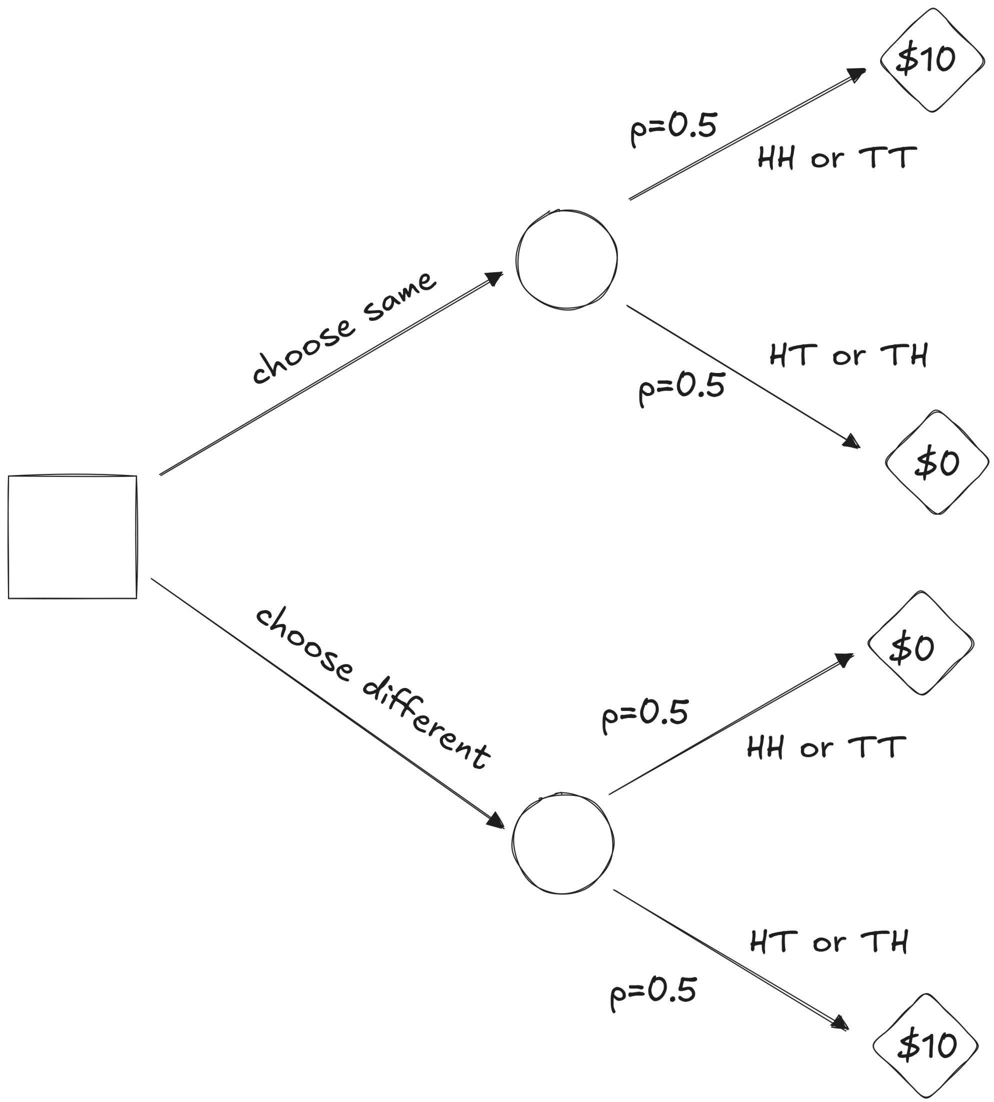
- How do we make uncertain decisions?
  - How do we consider uncertainty?



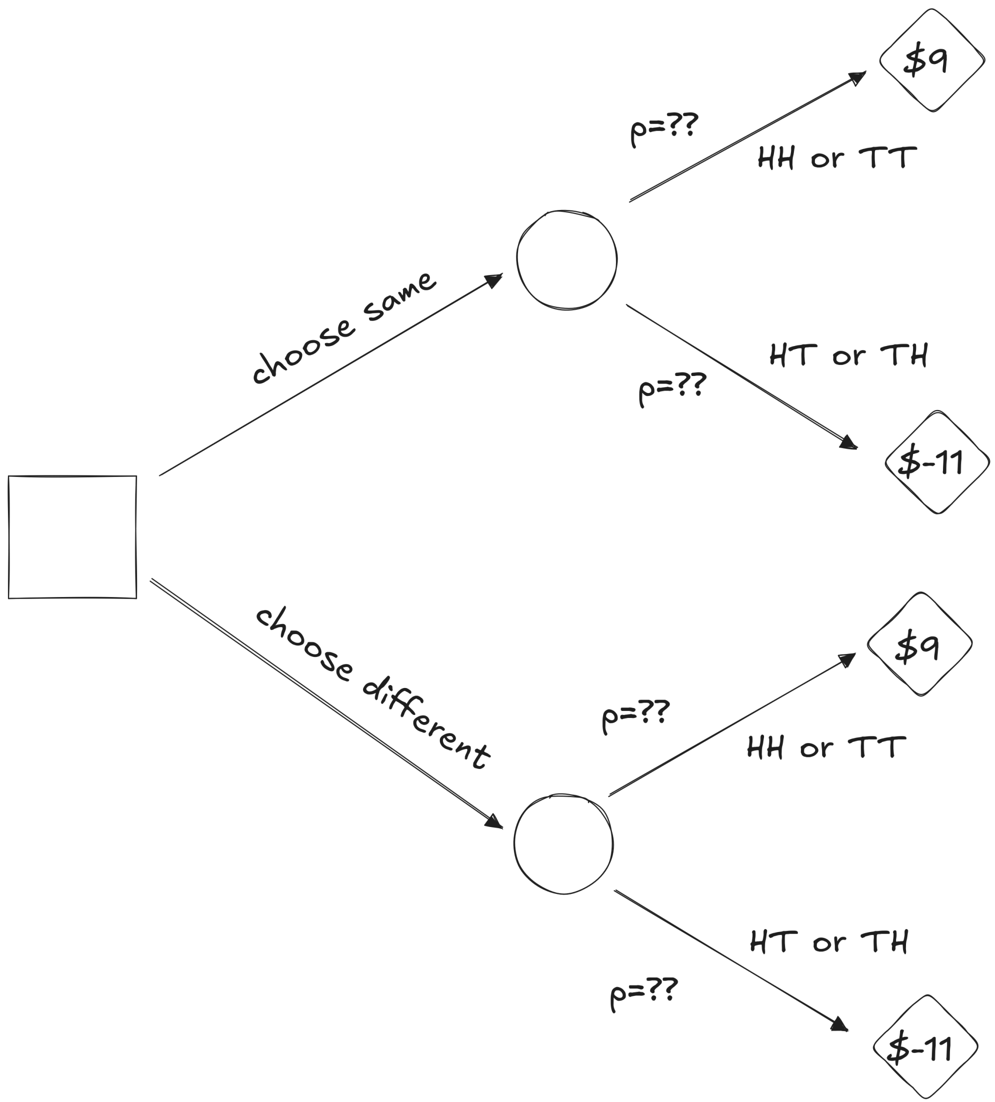
Decision


$10
p=0.5
H
flip one coin
T
p=0.5
$0
flip two coins
$24
p=0.25
HH
not HH
p=0.75
$0

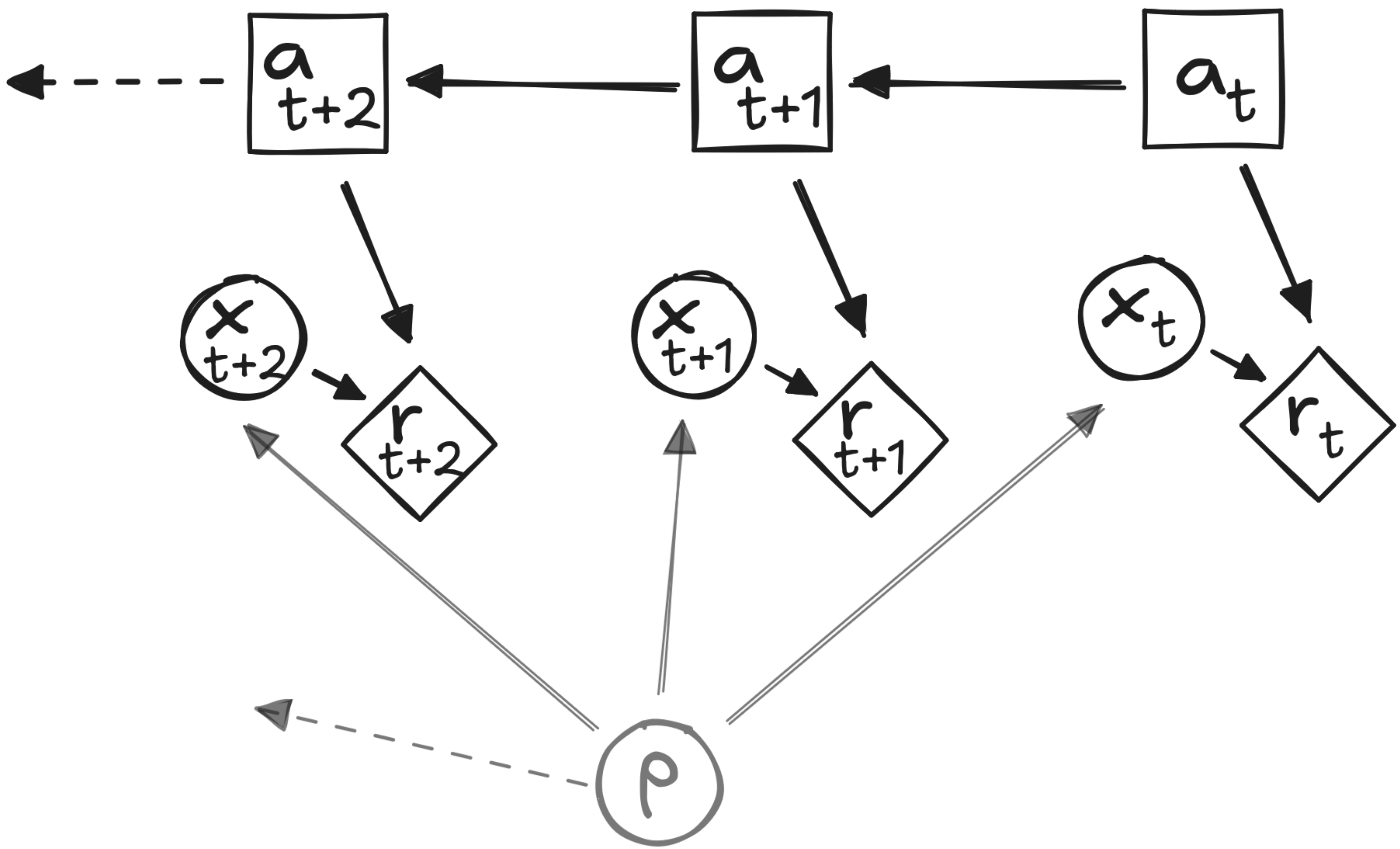
Games of Luck



Closer to Reality

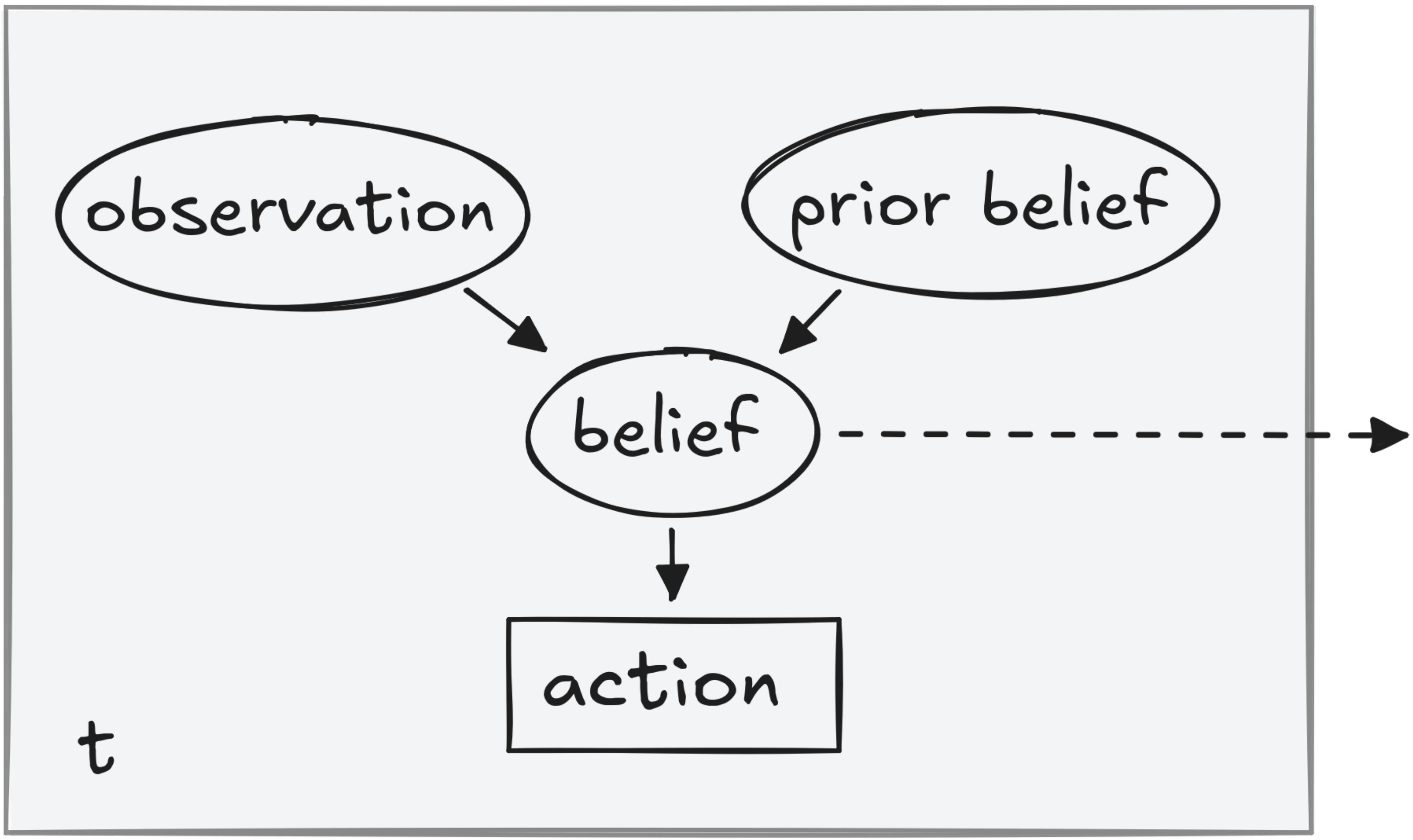


Belief



Updating Beliefs

Agent Function



Beliefs

- Parametric
  - Coin probability example
- Nonparametric
  - Particle filters

Need a world model.

Discrete State Filter

- Finite state space
- Finite observation space
- *Categorical* probability distributions
  - State is a vector
  - Belief state is a vector

Updating Beliefs

Observation model O

- $O(o|a, s')$
  - Probability of observing o given action a and transition to state s'
  - Part of our model
- We want: $P(s'|b, a, o)$

Updating Beliefs

$$P(s'|o,b,a) \propto P(o|s',b,a,) \cdot P(s'|b,a)$$

$$= O(o|a,s') \cdot P(s'|b,a)$$

$$= O(o|a,s') \sum_{s} P(s'|b,a,s)P(s|b,a)$$

$$= O(o|a,s') \sum_{s} T(s'|s,a)b(s)$$

Example

State 0: Sated State 1: Hungry

Action 0: Ignore State 1: Feed

$$T_{ignore} = \begin{bmatrix} 0.9 & 0.1 \\ 0 & 1 \end{bmatrix}$$

$$T_{feed} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$O(\text{quiet}|\text{sated}) = 0.9$$

$$O(\text{crying}|\text{sated}) = 0.1$$

$$O(\text{quiet}|\text{hungry}) = 0.2$$

$$O(\text{crying}|\text{hungry}) = 0.8$$

Continuous States

- Discrete state filtering impossible
  - Extended to continuous case
  - Summation becomes an integral
- We need to make assumptions
  - Linear gaussian assumption: Kalman Filter

Particle Filter

- Discretize continuous *belief state* space
  - State space can be continuous
  - Transition space can be continuous
  - Arbitrary dynamics

Particle Filter Details

- For each particle:
  - *Sample* result from transition model
- For each result:
  - *Weight* result by observation model
- From full result:
  - *Resample*

Decisions

Solving POMDPs

- POMDP $\rightarrow$ Belief-State MDP
- State space: all beliefs
- Action space: identical
- Reward space: identical

Belief state space is continuous.

Conditional Planning

- Plan is a “small” decision tree
  - Take an action
  - Observe next observation
  - Take subsequent actions based on observation

Conditional Plan - Example

Plan Utility

$$U^{\pi}(s) = R(s, \pi()) + \gamma \left[\sum_{s'} T(s' | s, \pi()) \sum_{o} O(o | \pi(), s') U^{\pi(o)}(s') \right]$$

- Can be evaluated recursively
- Finite horizon
- Tractable for small horizons
- Exponential explosion for larger horizons

Alpha Vectors

Expected utility of belief:

$$U^\pi(b) = \sum_s b(s) U^\pi(s)$$

As a vector:

$$U^\pi(b) = \sum_s b(s) U^\pi(s) = \boldsymbol{\alpha}_\pi^T \mathbf{b}$$

$\boldsymbol{\alpha}$ - expected utility under plan π for each state

Using Alpha Vectors

- Generate h-step conditional plans
- Calculate $Q(b, a)$
  - Compare with $Q(s, a)$ for MDPs
- Extract action

Plan Utility

Monte Carlo Tree Search

Multi-Armed Bandits

- Slot machine with more than one arm
- Each pull has a cost
- Each pull has a payout
- Probability of payouts unknown
- Goal: maximize reward
  - Time horizon?

Solving Multi-Armed Bandits

Confidence Bounds

- Expected value of reward per arm
  - Confidence interval of reward per arm
- Select arm based on upper confidence bound

- How do we estimate rewards?
  - Explore vs. exploit

Bandit as MDP?

Bandit Strategies

- Gittins Index: $\lambda = \max\limits_{T>0} \frac{E[\sum^{T-1} \gamma^t R_t]}{E[\sum^{T-1} \gamma^t]}$
- Upper Confidence Bound for arm $M_i:
  - $UCB(M_i) = \mu_i + \frac{g(N)}{\sqrt{N_i}}$
  - $g(N)$ is the "regret"
- Thompson Sampling
  - *Sample* arm based on probability of being optimal

Monte Carlo Methods

Tree Search

- Forget DFS, BFS, Dijkstra, A*
  - State space too large
  - Stochastic expansion
- Impossible to search entire tree
- Can *simulate* problem forward in time from starting state

Monte Carlo Tree Search

- Randomly simulate trajectories through tree
  - Complete trajectory
  - No heuristic needed[1]
  - *Need* a model
- Better than exhaustive search?

1. Heuristics can be used.

Selection Policy

- Focus search on “important” parts of tree
  - Similar to alpha-beta pruning
- Explore vs. exploit
  - Simulation
  - Not actually exploiting the problem
  - Exploiting the *search*

Monte Carlo Tree Search

- Choose a node
  - Explore/exploit
  - Choose a successor
  - Continue to leaf of search tree
- Expand leaf node
- Simulate result until completion
- Back-propagate results to tree

Monte Carlo Tree Search

Upper Confidence Bounds for Trees (UCT)

- MDP: Maximize $Q(s, a) + c\sqrt{\frac{\log N(s)}{N(s,a)}}$
  - Q for state s and action a
- POMDP: Maximize $Q(h, a) + c\sqrt{\frac{\log N(h)}{N(h,a)}}$
  - Q for history h and action a
  - History: action/observation sequence

Partially-Observable UCT

References

- Mykal Kochenderfer, Tim Wheeler, and Kyle Wray. *Algorithms for Decision Making.* 1st Edition, 2022.
- Richard S. Sutton and Andrew G. Barto. *Reinforcement Learning: An Introduction.* 2nd Edition, 2018.

David Silver and Joel Veness, Monte-Carlo Planning in Large POMDPs, *Advances in Neural Information Processing Systems 23 (NIPS 2010)*

- Stanford CS228 (Mykal Kochenderfer)