Review

CSCI 4511/6511

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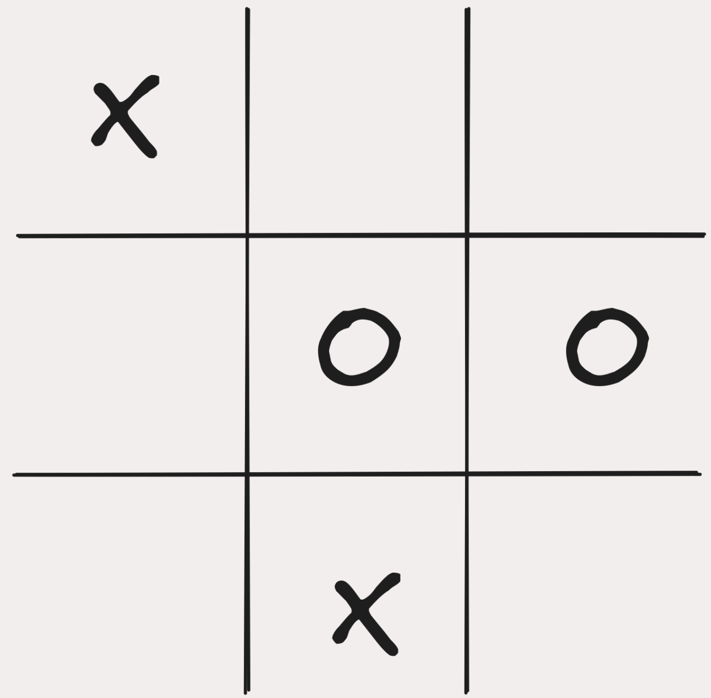
Announcements

- Extra Credit HW: Due 4 Dec
- **Project Proposals**
- Final Exam: 4 Dec
- Project Deadline: 13 Dec

Reflex Agent

- Very basic form of agent function
- Percept $\rightarrow$ Action lookup table
- Good for simple games
  - Tic-tac-toe
  - Checkers?
- Needs *entire state space* in table



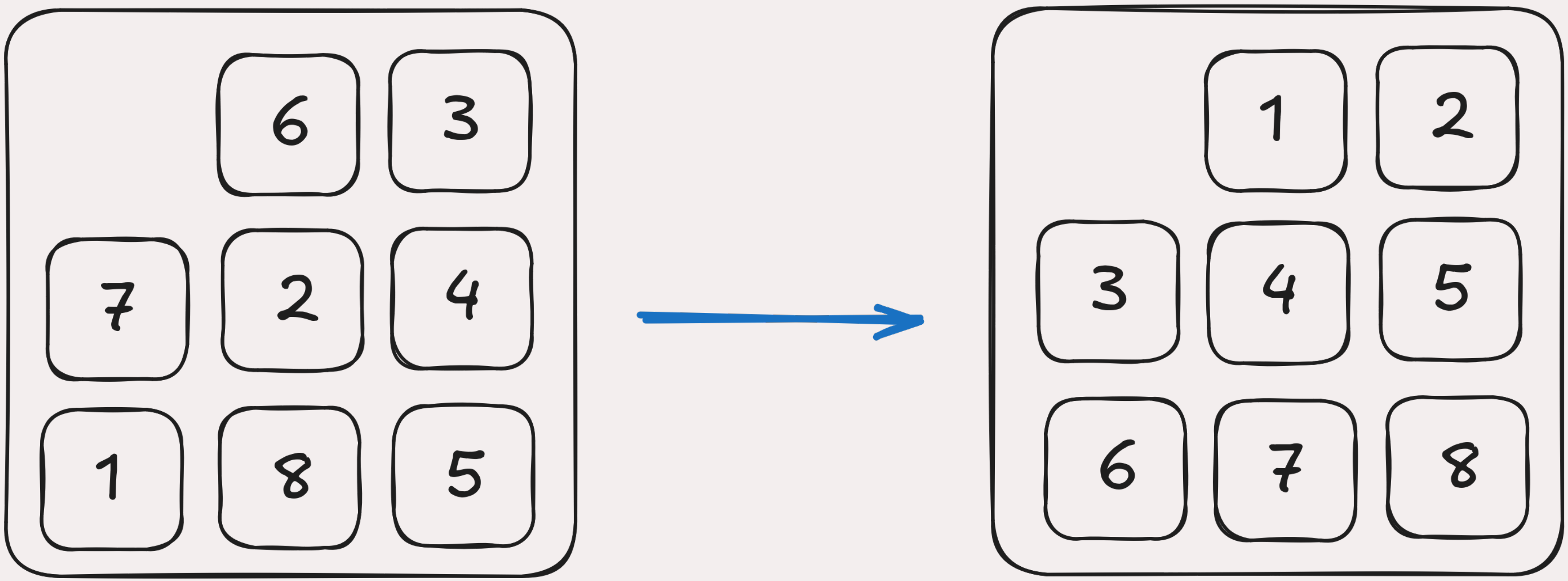
Partially-Observable State

- Most real-world problems
  - Sensor error
  - Model error
- Reflex agents fail[1]
- Agent needs a *belief state*

1. Unless total number of partial observations is bounded

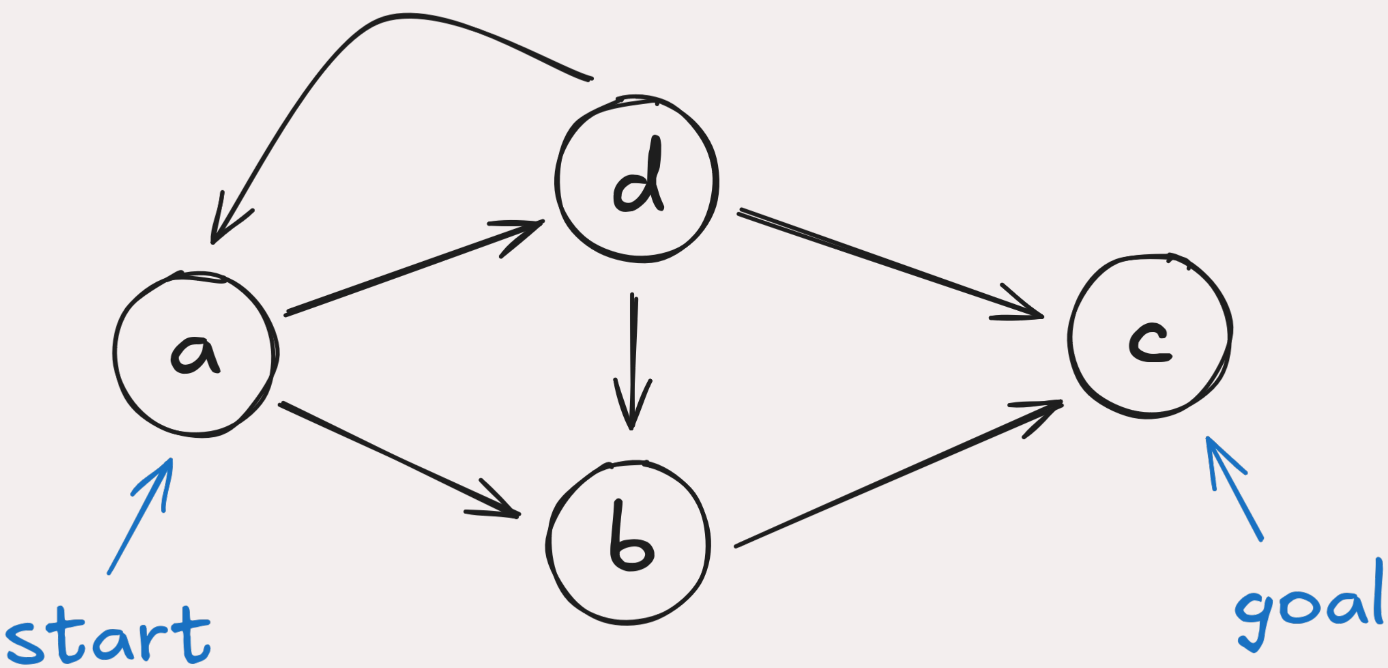
State

What is the state space?



Search: Why?

- Fully-observed problem
- Deterministic actions and state
- Well defined *start* and *goal*



Other Applications

- Route planning
- Protein design
- Robotic navigation
- Scheduling
  - Science
  - Manufacturing

Not Included

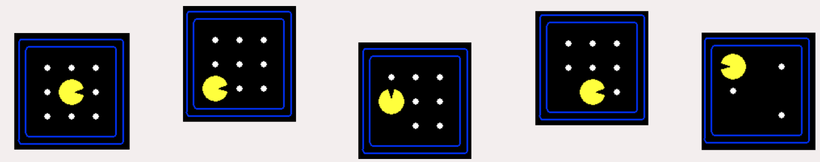
- Uncertainty
  - State transitions known
- Adversary
  - Nobody wants us to lose
- Cooperation
- Continuous state

Search Problem

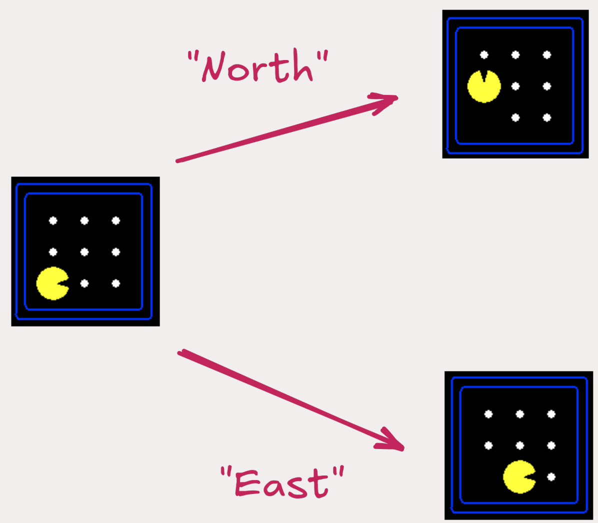
Search problem includes:

- Start State
- State Space
- State Transitions
- Goal Test

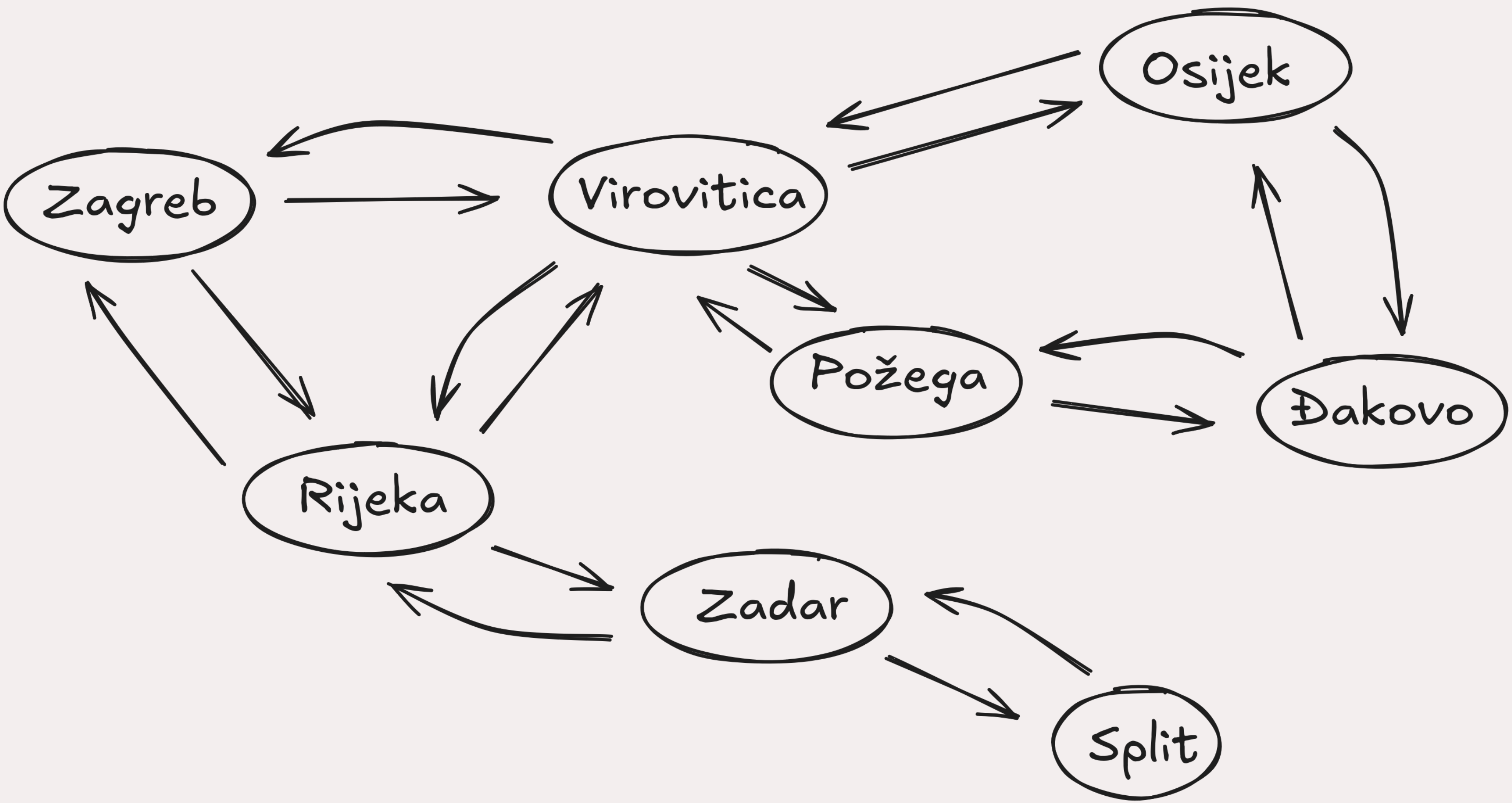
State Space:



Actions & Successor States:

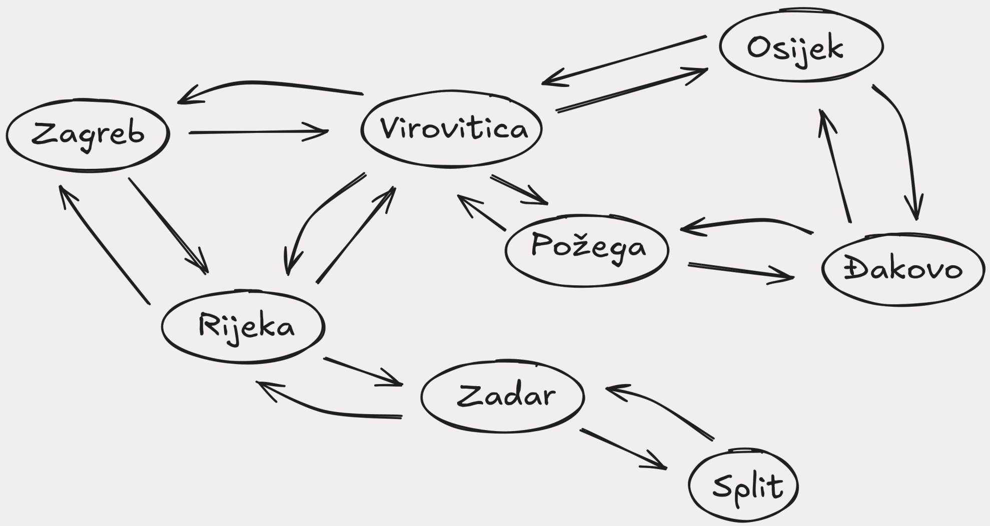


State Space Graph

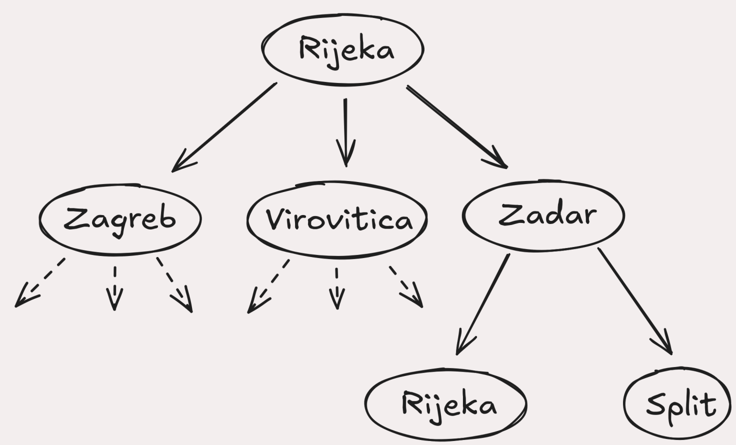


Search Trees

Graph:



Tree:



Let's Talk About Trees

- For any non-trivial problem, they're *big*
  - (Effective) branching factor
  - Depth
- Graph and tree both too large for memory
  - Successor function (graph)
  - Expansion function (tree)

How To Solve It

Given:

- Starting node
- Goal test
- Expansion

Do:

- Expand nodes from start
- Test each new node for goal
  - If goal, success
- Expand new nodes
  - If nothing left to expand, failure

Tree Search Algorithms

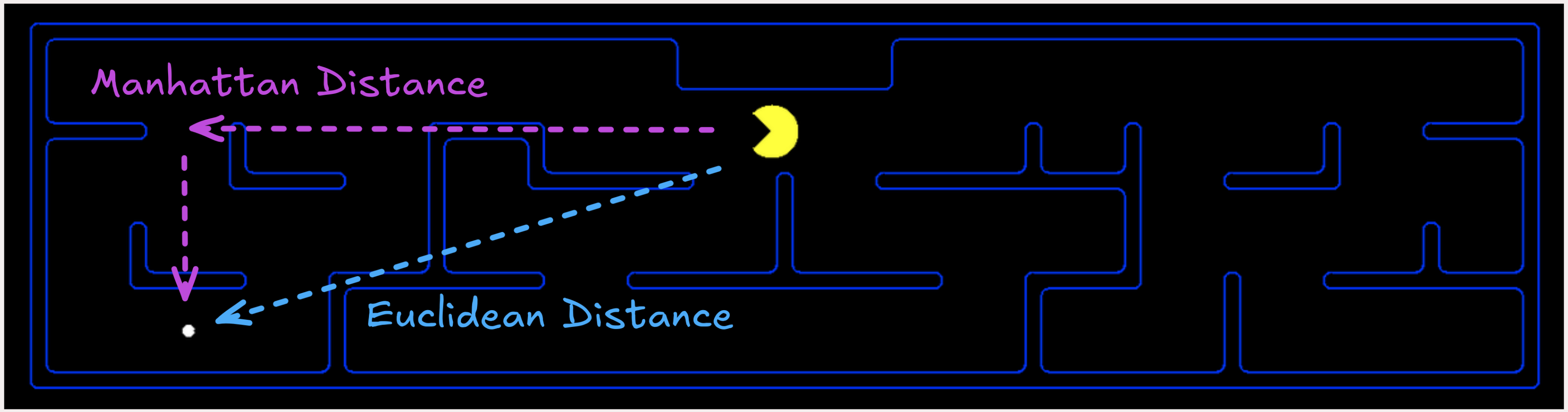
- BFS
- DFS
- UCS/Dijkstra
- A*
- Greedy searches

A* Search

- Include path-cost $g(n)$
  - $f(n) = g(n) + h(n)$
- Complete (always)
- Optimal (sometimes)
- Painful $O(b^m)$ time and space complexity

Choosing Heuristics

- Recall: $h(n)$ estimates cost from n to goal



- Admissibility
- Consistency

Choosing Heuristics

- Admissibility
  - *Never* overestimates cost from n to goal
  - Cost-optimal!
- Consistency
  - $h(n) \leq c(n, a, n') + h(n')$
  - n' successors of n
  - $c(n, a, n')$ cost from n to n' given action a

Consistency

- Consistent heuristics are admissible
  - Inverse not necessarily true
- Always reach each state on optimal path

Weighted A* Search

- Greedy: $f(n) = h(n)$
- A*: $f(n) = h(n) + g(n)$
- Uniform-Cost Search: $f(n) = g(n)$

...

- Weighted A* Search: $f(n) = W \cdot h(n) + g(n)$
  - Weight $W > 1$

Iterative-Deepening A* Search

"IDA*" Search

- Similar to Iterative Deepening with Depth-First Search
  - DFS uses depth cutoff
  - IDA* uses $h(n) + g(n)$ cutoff *with DFS*
  - Once cutoff breached, new cutoff:
    - Typically next-largest $h(n) + g(n)$
  - $O(b^m)$ time complexity 😔
  - $O(d)$ space complexity[1] 😌

1. This is slightly complicated based on heuristic branching factor b_h.

Where Do Heuristics Come From?

- Intuition
  - "Just Be Really Smart"
- Relaxation
  - The problem is constrained
  - Remove the constraint
- Pre-computation
  - Sub problems
- Learning

Local Search

Uninformed/Informed Search:

- Known start, known goal
- Search for optimal path

Local Search:

- “Start” is irrelevant
- Goal is not known
  - But we know it when we see it
- Search for *goal*

“Real-World” Examples

- Scheduling
- Layout optimization
  - Factories
  - Circuits
- Portfolio management
- Others?

Hill-Climbing

- Objective function
- State space mapping
  - Neighbors

Variations

- Sideways moves
  - Not free
- Stochastic moves
  - Full set
  - First choice
- Random restarts
  - If at first you don’t succeed, ~~you fail~~ try again!
  - Complete 😌

The Trouble with Local Maxima

- We don’t know that they’re local maxima
  - Unless we do?
- Hill climbing is efficient
  - But gets trapped
- Exhaustive search is complete
  - But it’s exhaustive!
  - Stochastic methods are ‘exhaustive’

Simulated Annealing

- Doesn't actually have anything to do with metallurgy
- Search begins with high "temperature"
  - Temperature decreases during search
- Next state selected randomly
  - Improvements always accepted
  - Non-improvements rejected stochastically
  - Higher temperature, less rejection
  - "Worse" result, more rejection

Local Beam Search

Recall:

- Beam search keeps track of k "best" branches

Local Beam Search:

- Hill climbing search, keeping track of k successors
  - Deterministic
  - Stochastic

Simple Games

- Two-player
- Turn-taking
- Discrete-state
- Fully-observable
- Zero-sum
  - This does some work for us!

Minimax

- Initial state s_0
- $\textsc{Actions}(s)$ and $\textsc{To-move}(s)$
- $\textsc{Result}(s, a)$
- $\textsc{Is-Terminal}(s)$
- $\textsc{Utility}(s, p)$

More Than Two Players

- Two players, two values: v_A, v_B
  - Zero-sum: $v_A = -v_B$
  - Only one value needs to be explicitly represented
- $>$ 2 players:
  - $v_A, v_B, v_C \ldots$
  - Value scalar becomes $\vec{v}$

Minimax Efficiency

Pruning removes the need to explore the full tree.

- Max and Min nodes alternate
- Once *one* value has been found, we can eliminate parts of search
  - Lower values, for Max
  - Higher values, for Min
- Remember highest value (α) for Max
- Remember lowest value (β) for Min

Solving Non-Deterministic Games

Previously: Max and Min alternate turns

Now:

- Max
- Chance
- Min
- Chance

😣

Expectiminimax

Constraint Satisfaction

- Express problem in terms of state variables
  - Constrain state variables
- Begin with all variables unassigned
- Progressively assign values to variables
- Assignment of values to state variables that "works:" *solution*

More Formally

- State variables: $X_1, X_2, \ldots, X_n$
- State variable domains: $D_1, D_2, \ldots, D_n$
  - The domain specifies which values are permitted for the state variable
  - Domain: set of allowable variables (or permissible range for continuous variables)[1]
  - Some constraints $C_1, C_2, \ldots, C_m$ restrict allowable values

1. Or a hybrid, such as a union of ranges of continuous variables.

Constraint Types

- Unary: restrict single variable
  - Can be rolled into domain
  - Why even have them?
- Binary: restricts two variables
- Global: restrict “all” variables

Constraint Examples

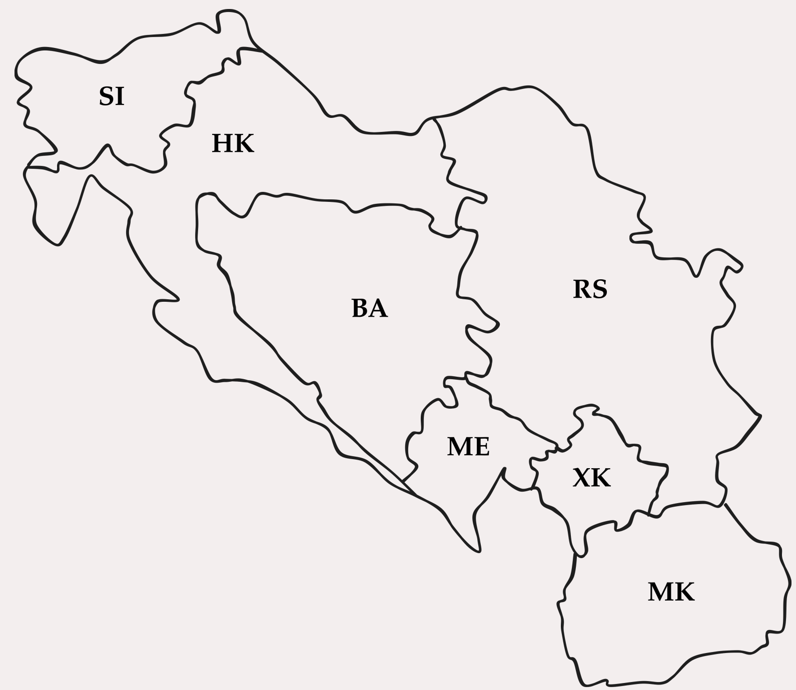
- X_1 and X_2 both have real domains, i.e. $X_1, X_2 \in \mathbb{R}$
  - A constraint could be $X_1 < X_2$
- X_1 could have domain $\{\text{red}, \text{green}, \text{blue}\}$ and X_2 could have domain $\{\text{green}, \text{blue}, \text{orange}\}$
  - A constraint could be $X_1 \neq X_2$
- $X_1, X_2, \ldots, X_100 \in \mathbb{R}$
  - Constraint: exactly four of X_i equal 12
  - Rewrite as binary constraint?

Assignments

- Assignments must be to values in each variable's domain
- Assignment violates constraints?
  - Consistency
- All variables assigned?
  - Complete

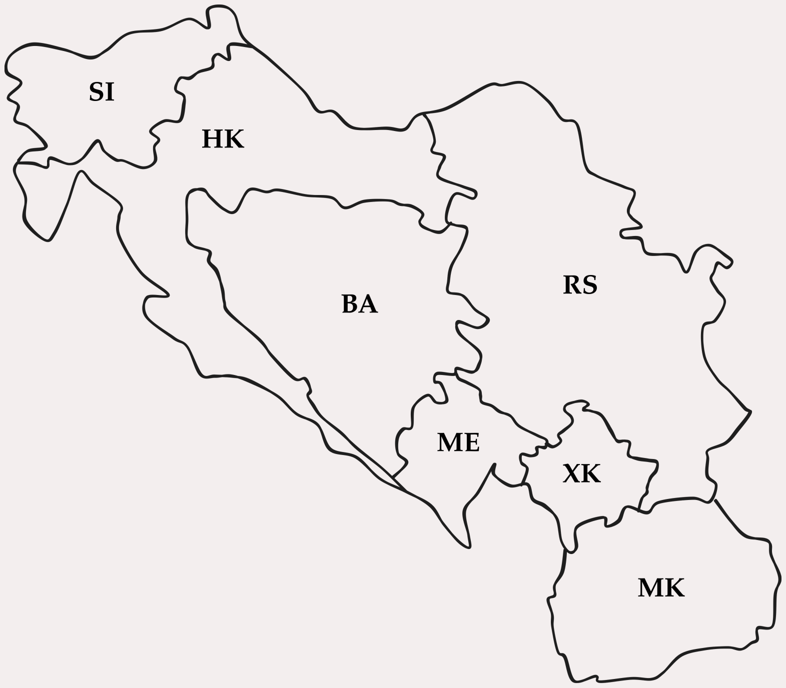
Graph Representation I

Constraint graph: edges are constraints



Graph Representation II

Constraint hypergraph: constraints are nodes



Solving CSPs

- We can search!
  - ...the space of consistent assignments
- Complexity $O(d^n)$
  - Domain size d, number of nodes n
- Tree search for node assignment
  - Inference to reduce domain size
- Recursive search

Inference

- Arc consistency
  - Reduce domains for pairs of variables
- Path consistency
  - Assignment to two variables
  - Reduce domain of third variable

Ordering

- SELECT-UNASSGINED-VARIABLE($CSP, assignment$)
  - Choose most-constrained variable[1]
- ORDER-DOMAIN-VARIABLES($CSP, var, assignment$)
  - Least-constraining value

- Tree-structure: *Linear time* solution

1. or MRV: "Minimum Remaining Values"

Logic

- $\neg$
  - "Not" operator, same as CS (`!`, `not`, etc.)
- $\wedge$
  - "And" operator, same as CS (`&&`, `and`, etc.)
  - This is sometimes called a *conjunction*.
- $\vee$
  - "Inclusive Or" operator, same as CS.
  - This is sometimes called a *disjunction*.

Unfamiliar Logical Operators

- $\Rightarrow$
  - Logical *implication*.
  - If $X_0 \Rightarrow X_1$, X_1 is always True when X_0 is True.
  - If X_0 is False, the value of X_1 is not constrained.
- $\iff$
  - "If and only If."
  - If $X_0 \iff X_1$, X_0 and X_1 are either both True or both False.
  - Also called a *biconditional*.

Knowledge Base & Queries

- We encode everything that we ‘know’
  - Statements that are true
- We query the knowledge base
  - Statement that we’d like to know about
- Logic:
  - Is statement consistent with KB?

Entailment

- $KB \models A$
  - "Knowledge Base entails A"
  - For every model in which KB is True, A is also True
  - One-way relationship: A can be True for models where KB is not True.
- Vocabulary: A is the *query*

Knowing Things

Falsehood:

- $KB \models \neg A$
  - No model exists where KB is True and A is True

It is possible to not know things:[1]

- $KB \not\models A$
- $KB \not\models \neg A$

1. $\not\models$ – "does not entail"

Conjunctive Normal Form

- *Literals* — symbols or negated symbols
  - X_0 is a literal
  - $\neg X_0$ is a literal
- *Clauses* — combine literals and disjunction using disjunctions ($\vee$)
  - $X_0 \vee \neg X_1$ is a valid disjunction
  - $(X_0 \vee \neg X_1) \vee X_2$ is a valid disjunction

Conjunctive Normal Form

- *Conjunctions* ($\wedge$) combine clauses (and literals)
  - $X_1 \wedge (X_0 \vee \neg X_2)$
- Disjunctions cannot contain conjunctions:
- $X_0 \vee (X_1 \wedge X_2)$ not in CNF
  - Can be rewritten in CNF: $(X_0 \vee X_1) \wedge (X_0 \vee X_2)$

Converting to CNF

- $X_0 \iff X_1$
  - $(X_0 \Rightarrow X_1) \wedge (X_1 \Rightarrow X_0)$
- $X_0 \Rightarrow X_1$
  - $\neg X_0 \vee X_1$
- $\neg(X_0 \wedge X_1)$
  - $\neg X_0 \vee \neg X_1$
- $\neg(X_0 \vee X_1)$
  - $\neg X_0 \wedge \neg X_1$

Joint Distributions

- Distribution over multiple variables
  - $P(x, y)$ represents $P\{X = x, Y = y\}$
- Marginal distribution:
  - $P(x) = \sum_y P(x, y)$

Independence

Conditional probability:

$$P(x|y) = \frac{P(x, y)}{P(y)}$$

Bayes' rule:

$$P(x|y) = \frac{P(y|x)P(x)}{P(y)}$$

Conditional Independence

$$P(x|y) = P(x) \rightarrow P(x, y) = P(x)P(y)$$

- Two variables can be conditionally independent…
  - … when conditioned on a third variable

Markov Chains

Markov property:

$$P(X_t | X_{t-1}, X_{t-2}, \ldots, X_0) = P(X_t | X_{t-1})$$

"The future only depends on the past through the present."

- State X_{t-1} captures "all" information about past
- No information in X_{t-2} (or other past states) influences X_t

State Transitions

Stochastic matrix P

$$P = \begin{bmatrix} P_{1,1} & \cdots & P_{1,n} \\ \vdots & \ddots & \\ P_{n,1} & & P_{n,n} \end{bmatrix}$$

- All rows sum to 1
- Discrete state spaces implied

Stationary Behavior

- "Long run" behavior of Markov chain

$x_0 P^k$ for large k

- "Stationary state" π such that:

$$\pi = \pi P$$

- Row eigenvector for P for eigenvalue 1
- 😌

Absorbing States

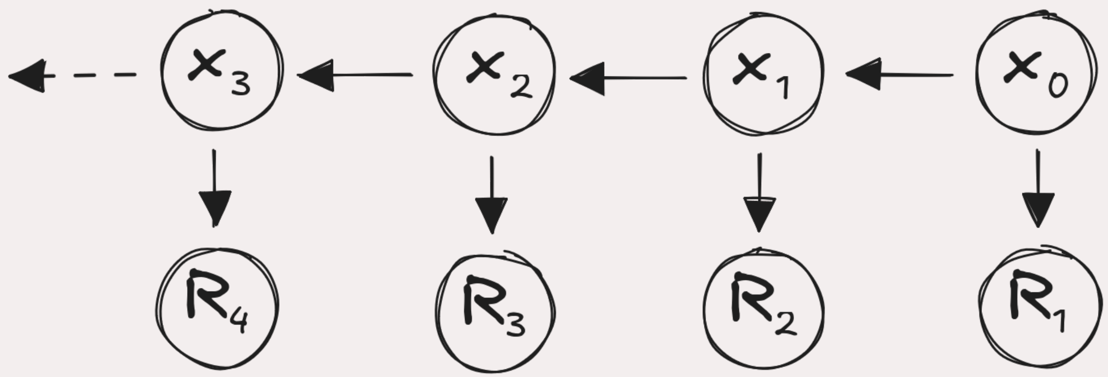
- State that cannot be "escaped" from
  - Example: gambling → running out of money

$$P = \begin{bmatrix} 0.5 & 0.3 & 0.1 & 0.1 \\ 0.3 & 0.4 & 0.3 & 0 \\ 0.1 & 0.6 & 0.2 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Non-absorbing states: "transient" states

Markov Reward Process

- Reward function $R_s = E[R_{t+1} | S_t = s]$:
  - Reward for being in state s
- Discount factor $\gamma \in [0, 1]$



$$U_t = \sum_k \gamma^k R_{t+k+1}$$

The Markov Decision Process

- Transition probabilities depend on actions

Markov Process:

$$s_{t+1} = s_t P$$

Markov Decision Process (MDP):

$$s_{t+1} = s_t P^a$$

Rewards: R^a with discount factor γ

MDP - Policies

- Agent function
  - Actions conditioned on states

$$\pi(s) = P[A_t = a | s_t = s]$$

- Can be stochastic
  - Usually deterministic
  - Usually *stationary*

MDP - Policies

State value function U^π:[1]

$$U^\pi(s) = E_\pi[U_t | S_t = s]$$

State-action value function Q^π:[2]

$$Q^\pi(s, a) = E_\pi[U_t | S_t = s, A_t = a]$$

Notation: E_π indicates expected value under policy π

1. Often simply called "value function"
2. Often simply called "action value function"

Bellman Expectation

Value function:

$$U^{\pi}(s) = E_{\pi}[R_{t+1} + \gamma U^{\pi}(S_{t+1})|S_t = s]$$

Action-value fuction:

$$Q^{\pi}(s,a) = E_{\pi}[R_{t+1} + \gamma Q^{\pi}(S_{t+1}, A_{t+1})|S_t = s, A_t = a]$$

Bellman Equation

$$U^*(s) = \max_a R(s,a) + \gamma \sum_{s'} T(s' | s,a) U^*(s')$$

Bellman Equation

$$Q^*(s,a) = R(s,a) + \gamma \sum_{s'} T(s'|s,a) \max_{a} Q^*(s',a')$$

How To Solve It

- No closed-form solution
    - *Optimal* case differs from policy evaluation

Iterative Solutions:

- Value Iteration
- Policy Iteration

Reinforcement Learning:

- Q-Learning
- Sarsa

Model Uncertainty

Action-value function:

$$Q(s,a) = R(s,a) + \gamma \sum_{s'} T(s'|s,a)U(s')$$

we don't know T:

$$U^\pi(s) = E_\pi\left[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \gamma^3 r_{t+3} + \ldots | s\right]$$

$$Q(s,a) = E_\pi\left[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \gamma^3 r_{t+3} + \ldots | s, a\right]$$

Temporal Difference (TD) Learning

- Take action from state, observe new state, reward

$$U(s) \leftarrow U(s) + \alpha \left[r + \gamma U(s') - U(s) \right]$$

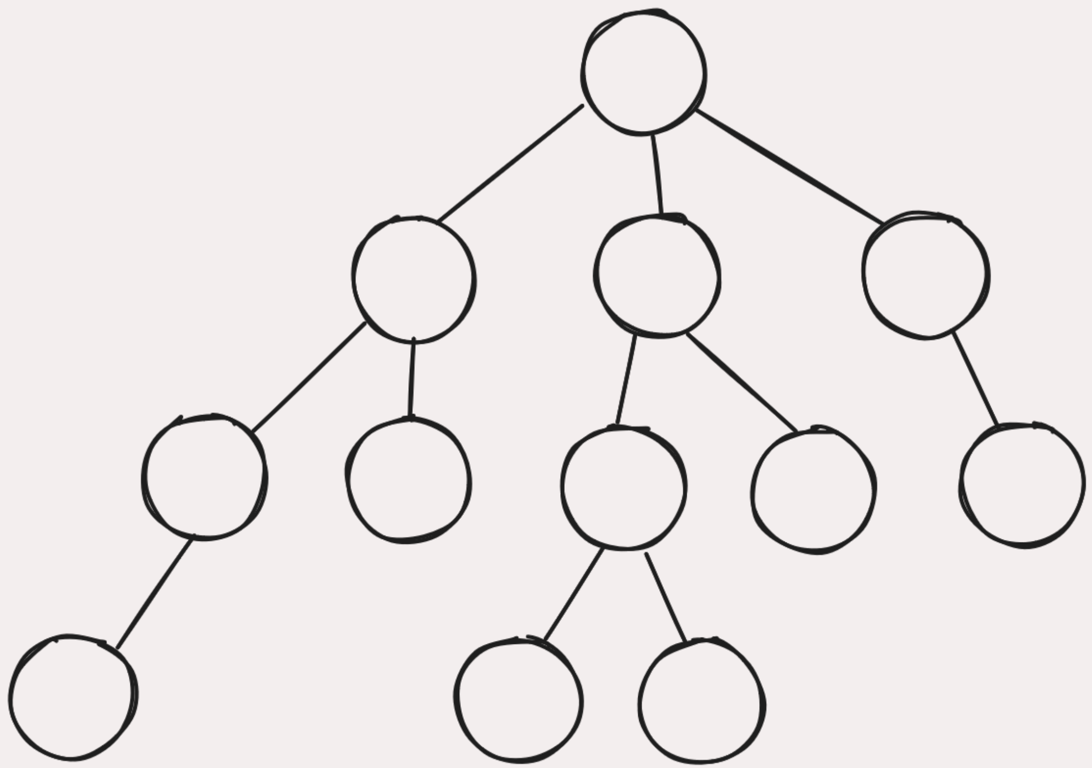
- Update immediately given (s, a, r, s')

- TD Error: $\left[r + \gamma U(s') - U(s) \right]$
  - Measurement: $r + \gamma U(s')$
  - Old Estimate: $U(s)$

Methods

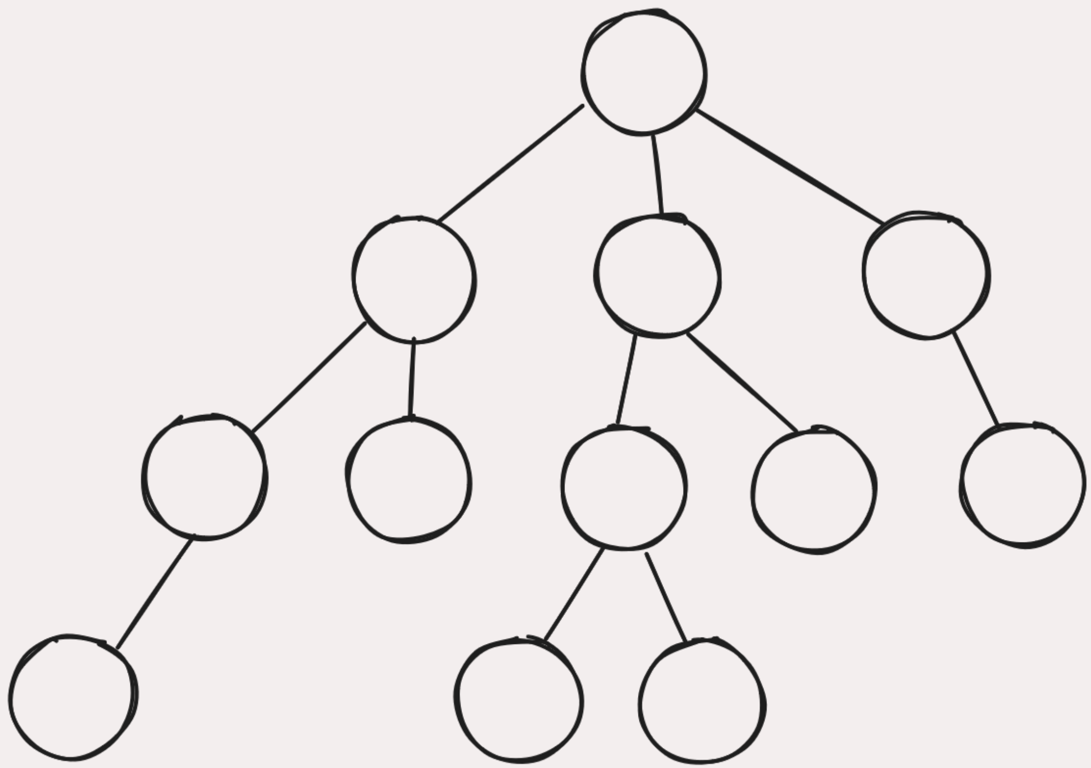
- Q-Learning
- Sarsa
- Eligibility traces
- Local approximation

Monte Carlo Tree Search - Search



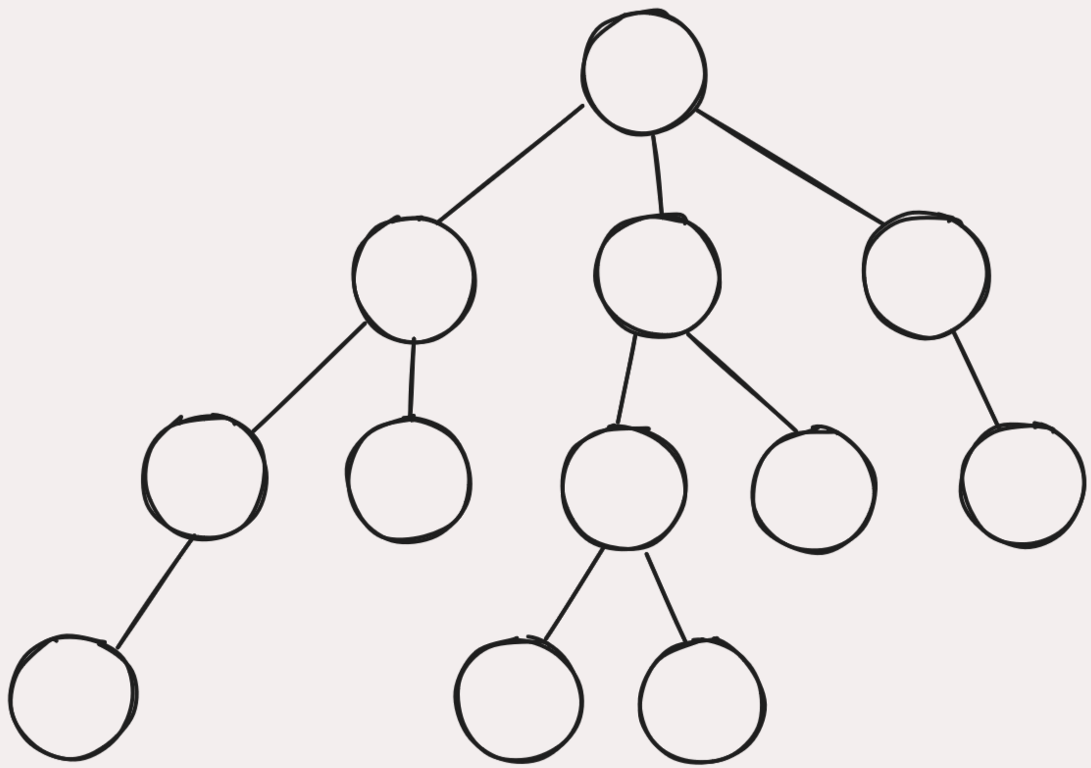
- If current state $\in T$ (tree states):
  - Maximize: $Q(s,a) + c\sqrt{\frac{\log N(s)}{N(s,a)}}$
  - Update $Q(s,a)$ during search

Monte Carlo Tree Search - Expansion



- State $\notin T$
  - Initialize $N(s, a)$ and $Q(s, a)$
  - Add state to T

Monte Carlo Tree Search - Rollout



- Policy π_0 is "rollout" policy
  - Usually stochastic
  - States *not* tracked

References

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