This is ungraded examination preparation.

CSCI 4511/6511 - Exam Prep 4

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Instructions:
This is exam and prep and is not for credit. You should try to complete it on your own, without references.

1 Markov Processes

The Ignatius Coffee Company in Cleric, AZ, has two espresso machines and two baristas. It takes about five minutes for one barista to brew one cup of coffee. There is room for four people total to wait for their coffee.

- For this problem, model each time step as five minutes in length.
- If a customer enters during a time step *and* an espresso machine is free:
  - The customer's coffee is brewed during that time step
- Otherwise, if a customer enters during at time step and no machine is free:
  - The customer waits
- In any time step, there is an equal probability that 0, 1, 2, or 3 customers will arrive.

Ignatius himself has decreed that, after 5 PM, the shop will stay open until no more customers are waiting, after which the shop will close (and no more customers will be accepted to queue for coffee).

Model the number of customers waiting for coffee as a Markov chain.

2 Stationary Markov Processes

During the day (from 8 AM until 5 PM), there is an equal probability that 0, 1, 2, 3, 4, or 5 customers will arive in any five-minute step. After 5 PM, the probabilities for customer arrival are the same as the previous problem.

What is the probability distrubution for the number of customers waiting at the shop at 5 PM? Model the problem as a Markov chain and explain how you would solve it– you do **not** need to compute a solution, but your solution must explain every step of the computation.

3 Markov Reward Processes

The coffee shop nets $1 per cup of coffee served (after expenses). Assuming no other changes in the business, how much do profits rise if a fifth waiting spot is added?

Model the problem as a Markov reward process and explain how you would solve it– you do **not** need to compute a solution, but your solution must explain every step of the computation.

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