Search

CSCI 4511/6511

Joe Goldfrank

Good Afternoon

- Good afternoon

Announcements

- Homework 1 is due on 7 February at 11:55 PM
  - Automatic extensions
  - Pay attention!

Why Are We Here?

- We're designing rational agents!
  - Perception
  - Logic
  - Action

In Practice

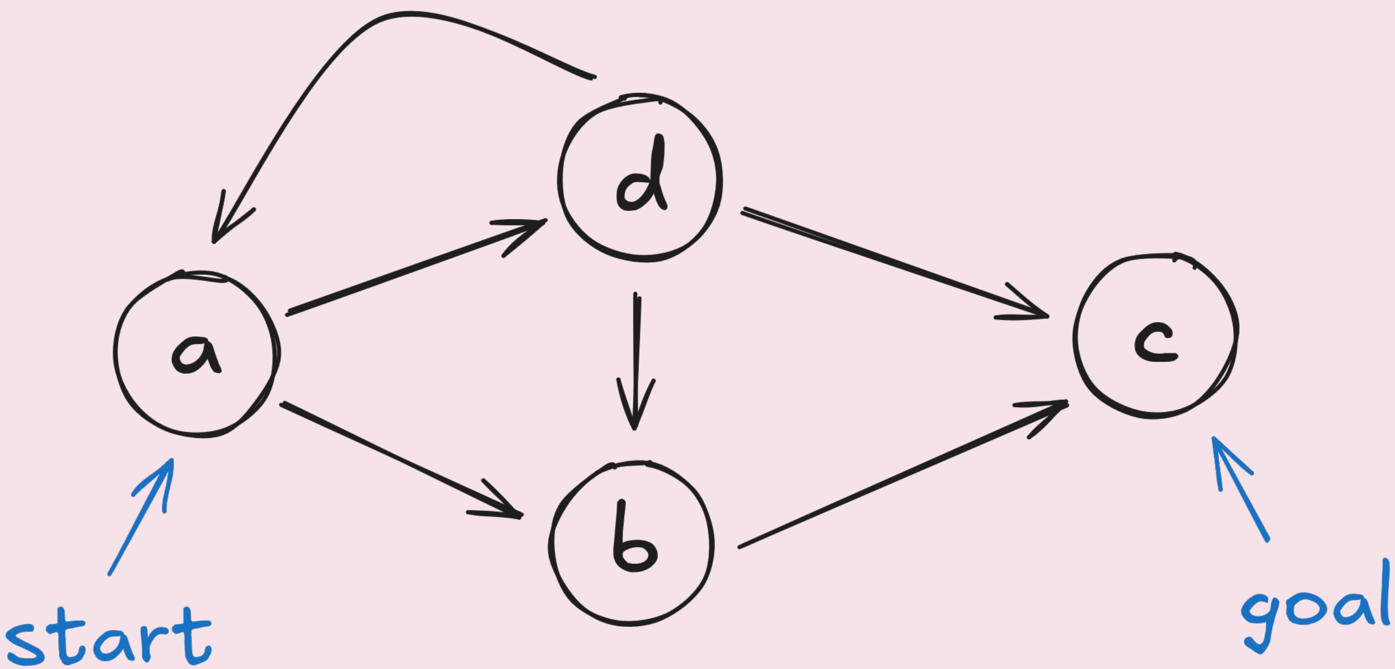
- Environment
  - What happens next
- Perception
  - What agent can see
- Action
  - What agent can do
- Measure/Reward
  - Encoded utility function

Reframed

- Building a model of the real world
  - Model is based on sensor inputs
  - Model is flawed
- Solve problems *on the model*
  - Take actions based on solution
- Model close to reality → solution useful
  - Else: 🫠

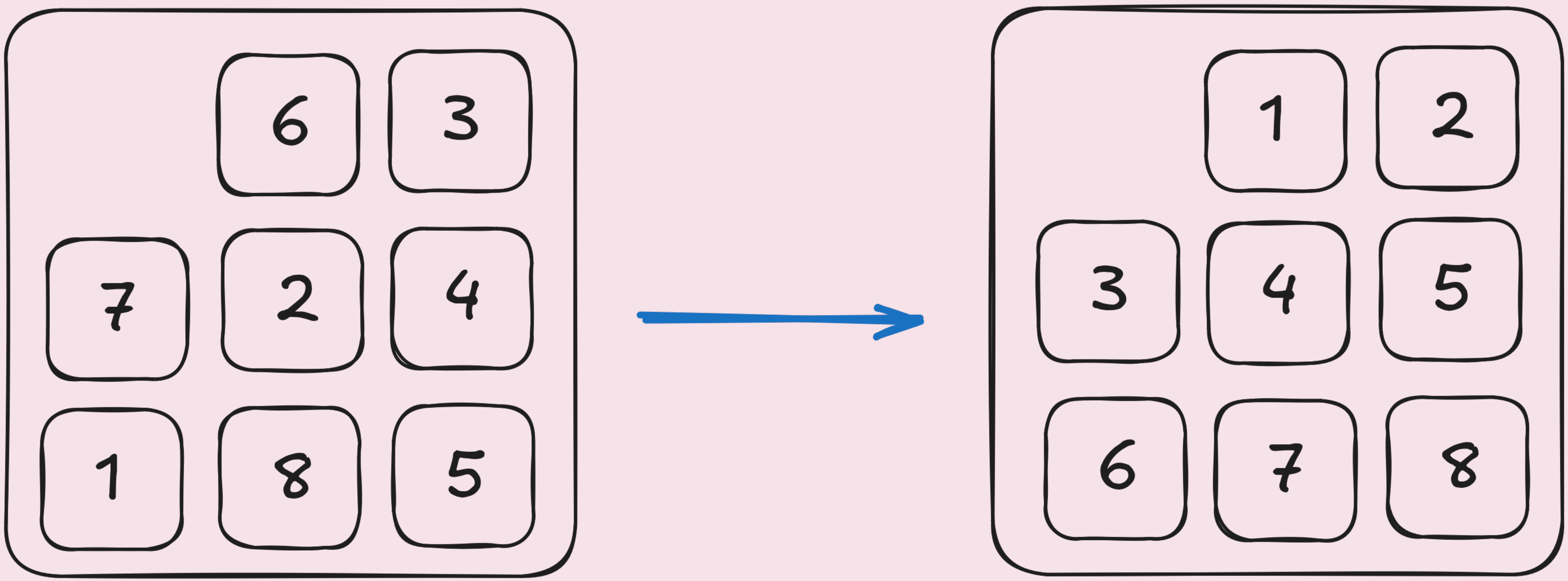
Search: Why?

- Fully-observed problem
- Deterministic actions and state
- Well defined *start* and *goal*



State

What is the state space?



State

RD
Glenmont
Wheaton
Forest Glen
Silver Spring
Takoma
Fort Totten
Brookland-CUA
Rhode Island Ave
NoMa-Gallaudet U
Union Station
Judiciary Sq
Shady Grove
Rockville
Twinbrook
North Bethesda
Grosvenor-Strathmore
Medical Center
Bethesda
Friendship Heights
Tenleytown-AU
Van Ness-UDC
Cleveland Park
Woodley Park
Dupont Circle
Farragut North
Metro Center
Gallery Place
GR
Greenbelt
College Park-U of Md
Hyattsville Crossing
West Hyattsville
Georgia Ave-Petworth
Columbia Heights
U St
Shaw-Howard U
Mt Vernon Sq
Archives
L'Enfant Plaza
Waterfront
Navy Yard-Ballpark
Anacostia
Congress Heights
Southern Ave
Naylor Rd
Suitland
Branch Ave
SV
Ashburn
Loudoun Gateway
Washington Dulles International Airport
Innovation Center
Herndon
Reston Town Center
Wiehle-Reston East
Spring Hill
Greensboro
Tysons
McLean
OR
Vienna
Dunn Loring
West Falls Church
East Falls Church
Ballston-MU
Virginia Sq-GMU
Clarendon
Court House
Rosslyn
Foggy Bottom-GWU
Farragut West
McPherson Sq
Federal Triangle
Smithsonian
Federal Center SW
Capitol South
Eastern Market
Potomac Ave
Stadium-Armory
Minnesota Ave
Deanwood
Cheverly
Landover
New Carrollton
Benning Rd
Capitol Heights
Addison Rd
Morgan Blvd
Downtown Largo
BL
Arlington Cemetery
Pentagon
Pentagon City
Crystal City
Ronald Reagan Washington National Airport
Potomac Yard
Braddock Rd
King St-Old Town
Van Dorn St
Franconia-Springfield
YL
Eisenhower Ave
Huntington
Metro is accessible.
N
Map is not to scale

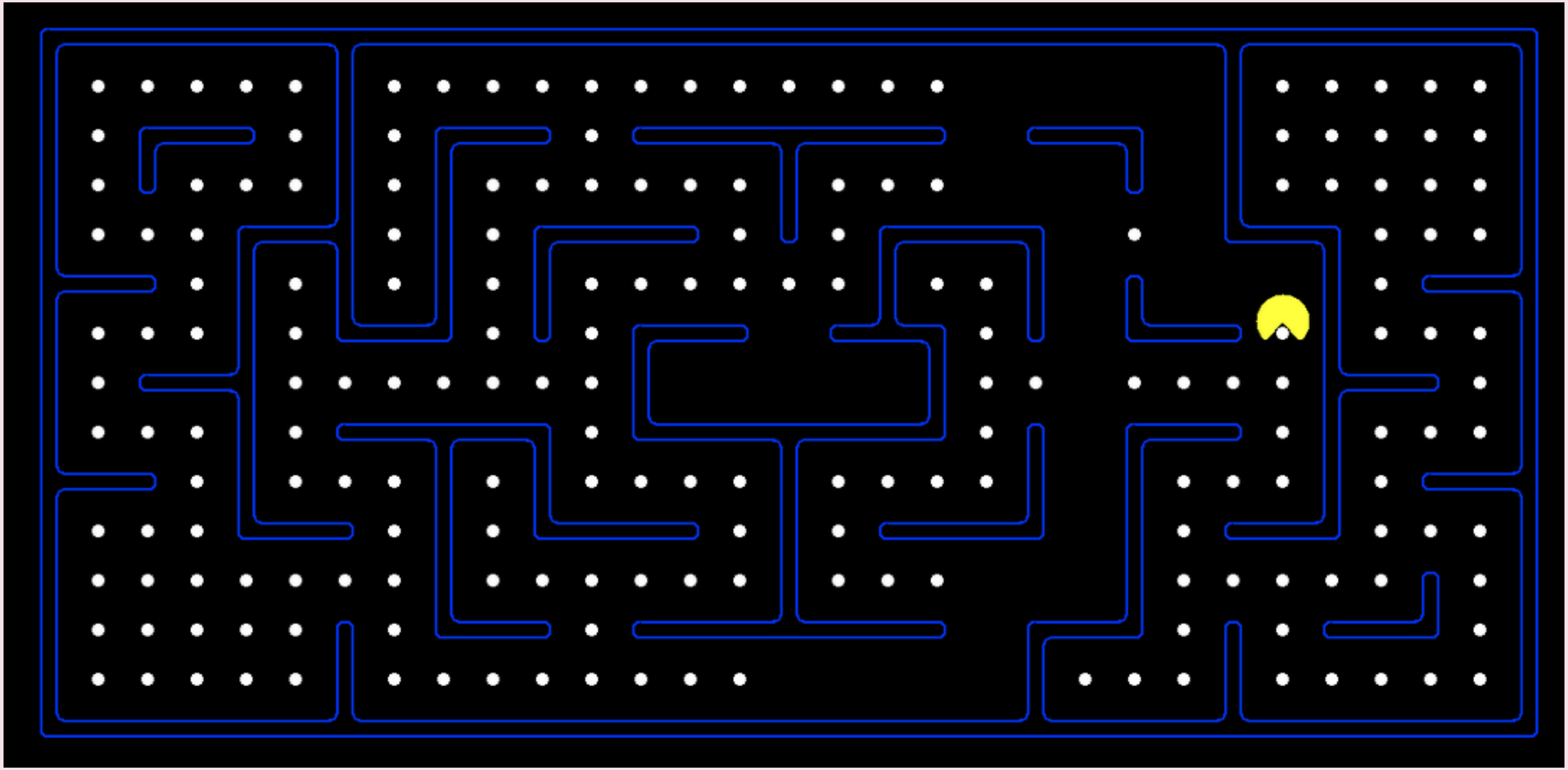
Other Applications

- Route planning
- Protein design
- Robotic navigation
- Scheduling
  - Science
  - Manufacturing

Not Included

- Uncertainty
  - State transitions known
- Adversary
  - Nobody wants us to lose
- Cooperation
- Continuous state

Who Is The Pac-Man?

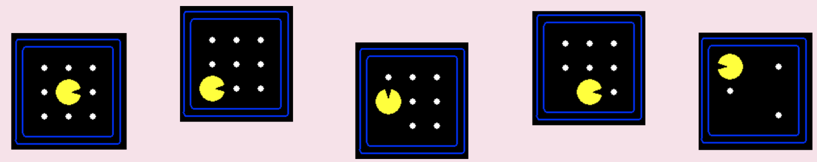


Search Problem

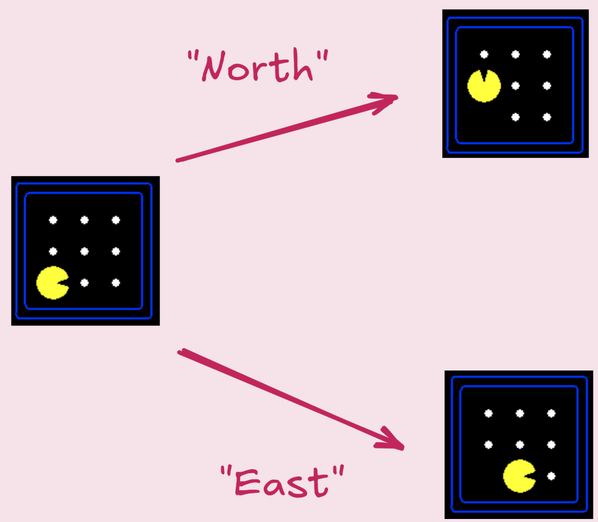
Search problem includes:

- Start State
- State Space
- State Transitions
- Goal Test

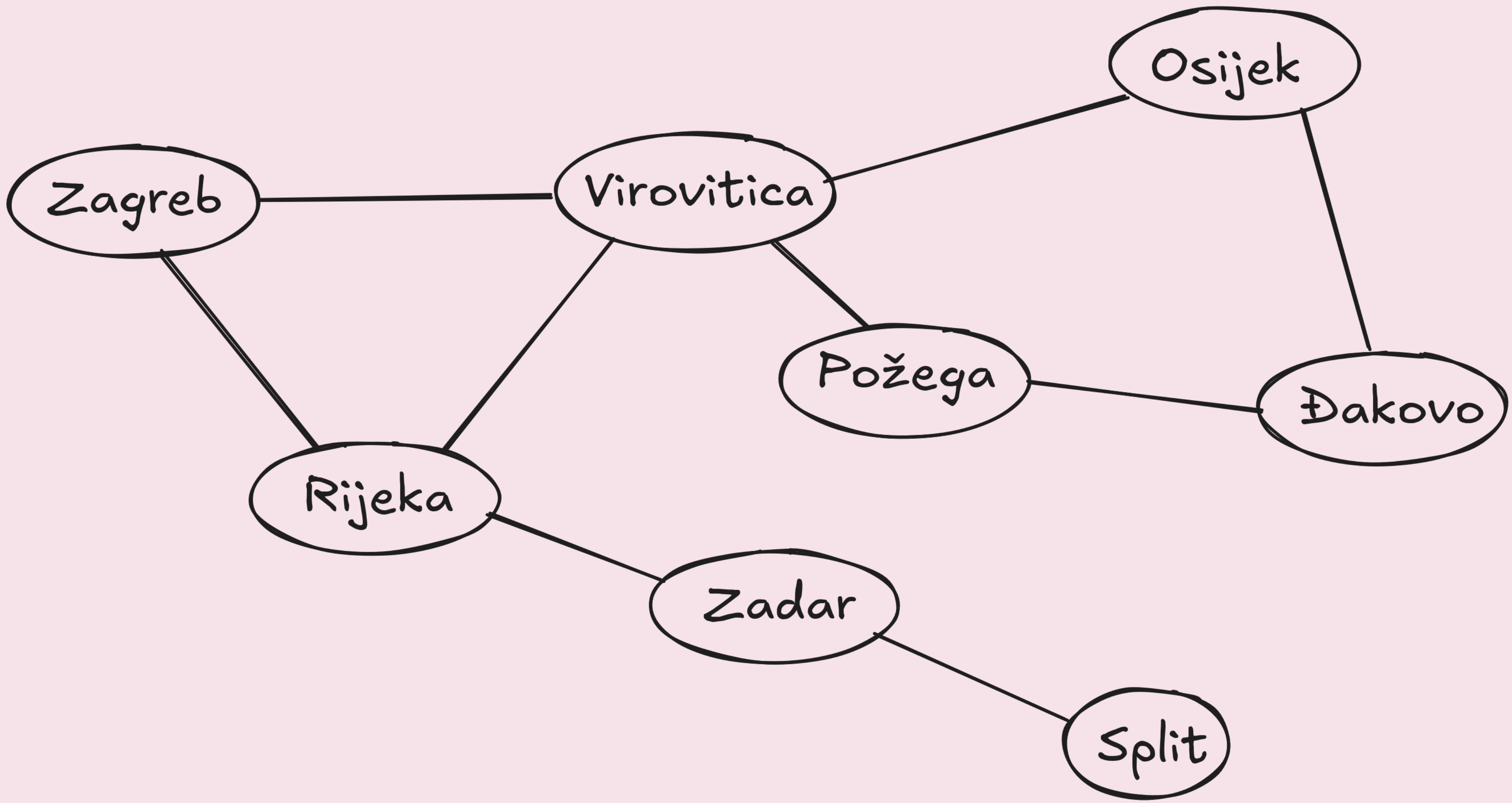
State Space:



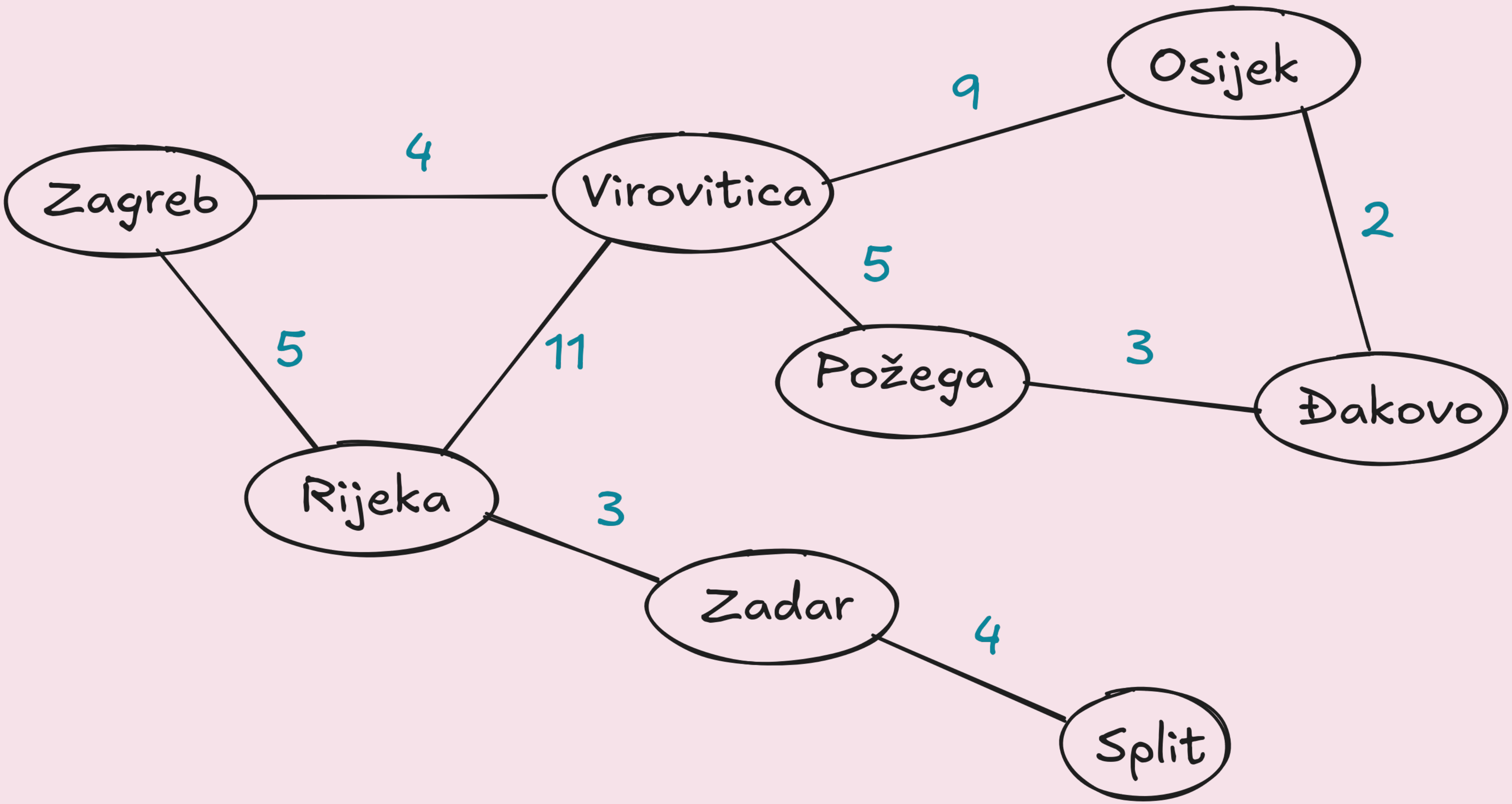
Actions & Successor States:



Tour of Croatia

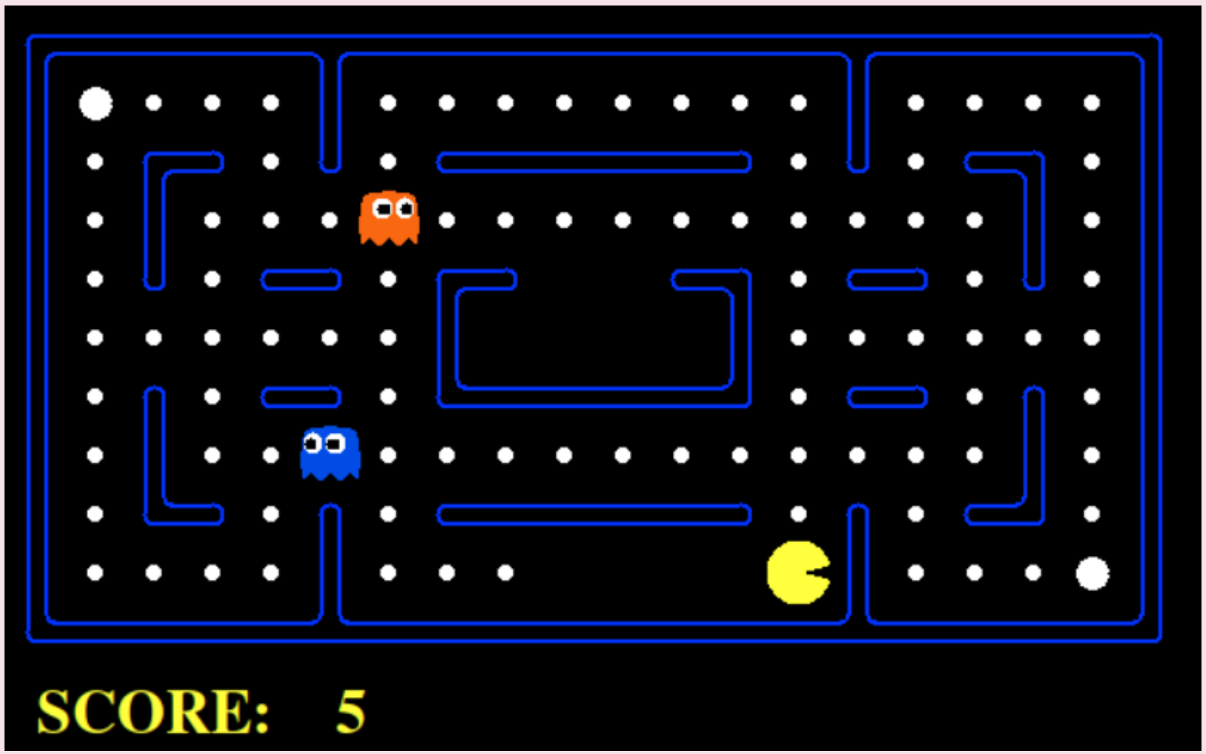


Tour of Croatia

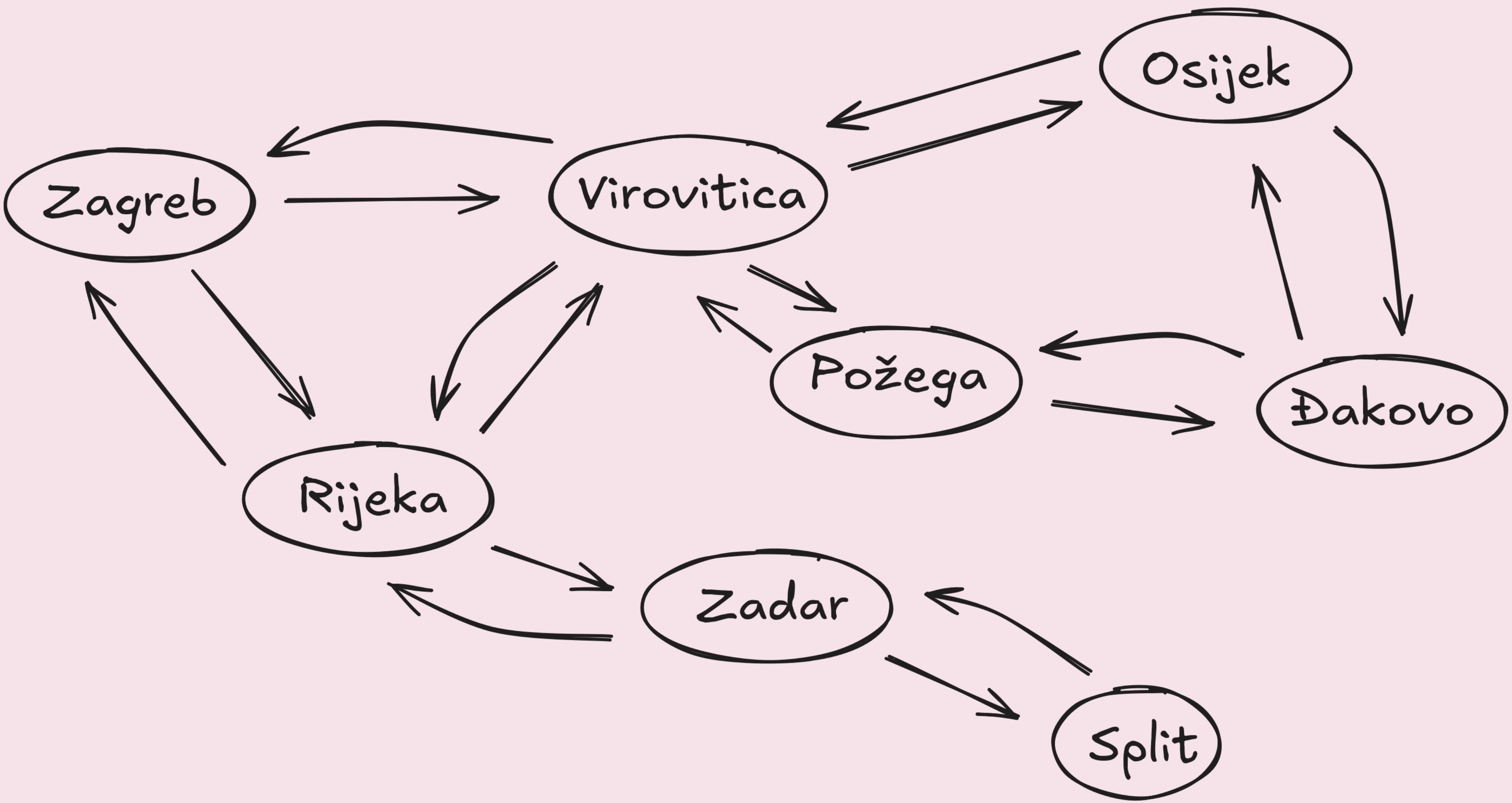


State Space Size?

- Pacman positions, Wall Positions
- Food positions, Food Status?
- Ghost positions, Ghost Status?

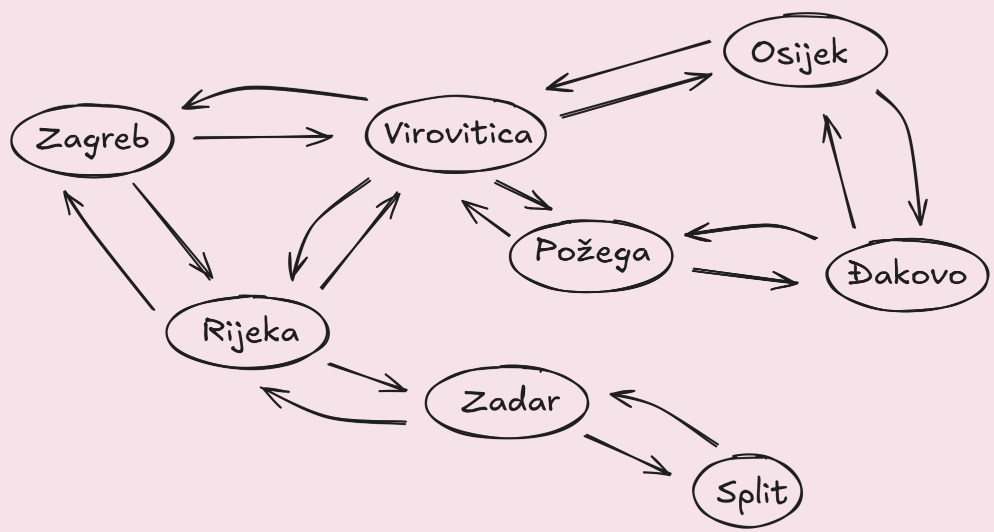


State Space Graph

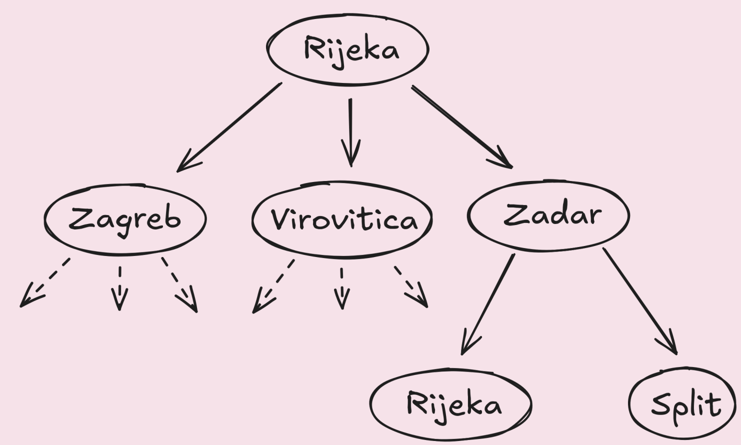


Search Trees

Graph:

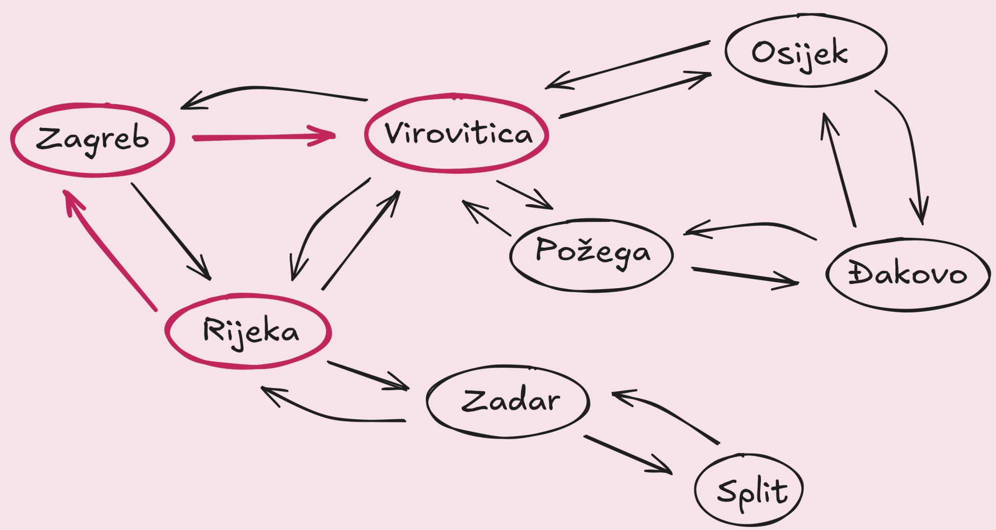


Tree:

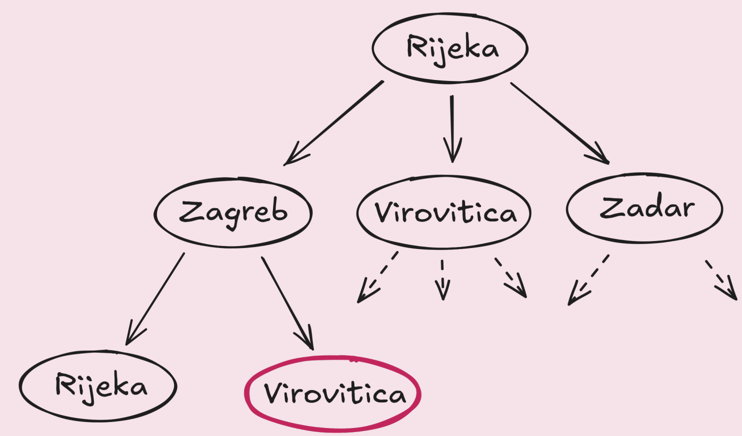


Node Representation

Graph:



Tree:



Let's Talk About Trees

- For any non-trivial problem, they're *big*
  - (Effective) branching factor
  - Depth
- Graph and tree both too large for memory
  - Successor function (graph)
  - Expansion function (tree)

How To Solve It

Given:

- Starting node
- Goal test
- Expansion

Do:

- Expand nodes from start
- Test each new node for goal
  - If goal, success
- Expand new nodes
  - If nothing left to expand, failure

Best-First Search

Algorithm Best-First Search

```
function BEST-FIRST-SEARCH(problem, f)
    node ← NODE(STATE=problem.INITIAL)
    frontier ← priority queue ordered by f
    frontier.ADD(node)
    reached ← lookup table
    reached[node] ← problem.INITIAL
    while not IS-EMPTY(frontier) do
        node ← POP(frontier)
        if problem.IS-GOAL(node.STATE) then
            return node
        for each child in EXPAND(problem,node) do
            s ← child.STATE
            if not s ∈ reached or child.PATH-COST < reached[s].PATH-COST then
                reached[s] ← child
                frontier.ADD(child)
    return failure

function EXPAND(problem, node)
    s ← node.STATE
    for each action in problem.ACTIONS(s) do
        s′ ← problem.RESULT(s, action)
        cost ← node.PATH-COST +problem.ACTION-COST(s, action, s′)
        yield NODE(STATE= s′, PARENT=node, ACTION=action, PATH-COST=cost)
```

Frontier Expansion

Frontier Expansion

- Frontier: nodes “currently” expanded
  - If no frontier node is goal, need to add to frontier
  - How?
- Can we have cycles?
  - How do we deal with cycles?

Queues & Searches

- Priority Queues
  - Best-First Search
  - Uniform-Cost Search[1]
- FIFO Queues
  - Breadth-First Search
- LIFO Queues[2]
  - Depth-First Search

1. Also known as “Dijkstra’s Algorithm,” because it is Dijkstra’s Algorithm
2. Also known as “stacks,” because they are stacks.

Search Features

- Completeness
  - If there is a solution, will we find it?
- Optimality
  - Will we find the *best* solution?
- Time complexity
- Memory complexity

Breadth-First Search

- FIFO Queue
- Complete
- Optimal
- $O(b^d)$
- Nice features for equal-weight arcs:
  - Lowest-cost path first
  - *reached* collection can be a set

Breadth-First Search

Algorithm Breadth-First Search

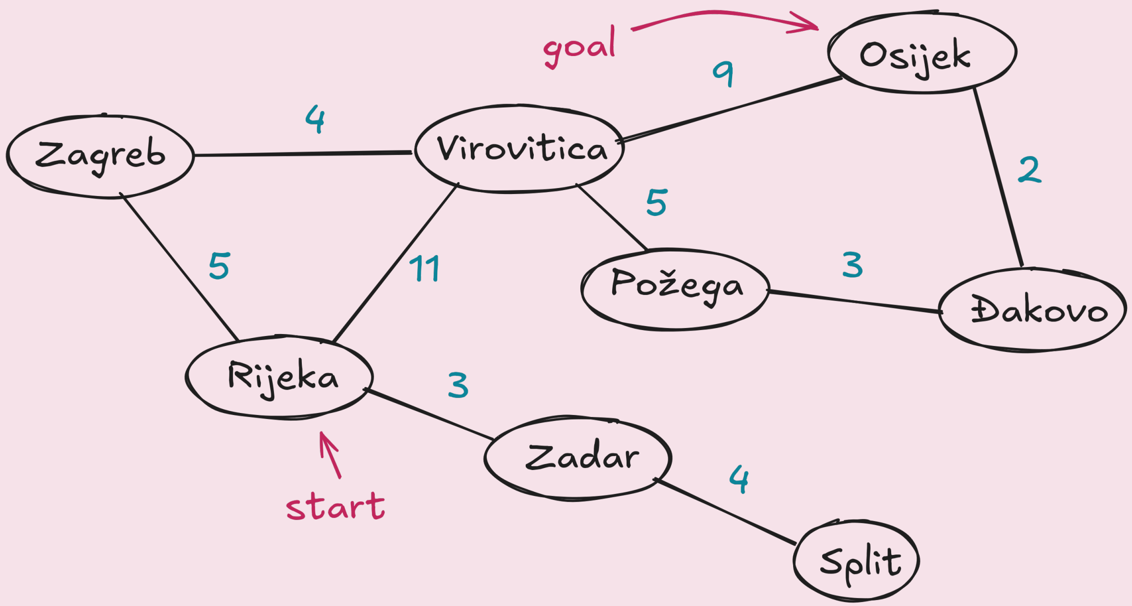
```
function BREADTH-FIRST-SEARCH(problem)
    node ← NODE(STATE=problem.INITIAL)
    if problem.IS-GOAL(node.STATE) then
        return node
    frontier ← FIFO queue
    frontier.ADD(node)
    reached ← set
    reached ← problem.INITIAL
    while not IS-EMPTY(frontier) do
        node ← POP(frontier)
        for each child in EXPAND(problem,node) do
            s ← child.STATE
            if problem.IS-GOAL(s) then
                return child
            if not s ∈ reached then
                reached.ADD(child)
                frontier.ADD(child)
    return failure
```

Uniform-Cost Search

Non-uniform costs → BFS inappropriate.

Algorithm Uniform-Cost Search

```
function UNIFORM-COST-SEARCH(problem)
    return BEST-FIRST-SEARCH(problem, PATH-COST)
```



Depth-First Search

- "Family" of searches
- LIFO stack
- Problems?

Algorithm Depth-First Search

```
function DEPTH-FIRST-SEARCH(problem)
    node ← NODE(STATE=problem.INITIAL)
    frontier ← LIFO stack
    frontier.PUSH(node)
    while not IS-EMPTY(frontier) do
        node ← POP(frontier)
        if problem.IS-GOAL(node.STATE) then
            return node
        else if not IS-CYCLE(node) then
            for each child in EXPAND(problem,node) do
                frontier.PUSH(child)
    return failure
```

Uninformed Search Variants

- Depth-Limited Search
  - Fail if depth limit reached (why?)
- Iterative deepening
  - vs. Breadth-First Search
- Bidirectional Search

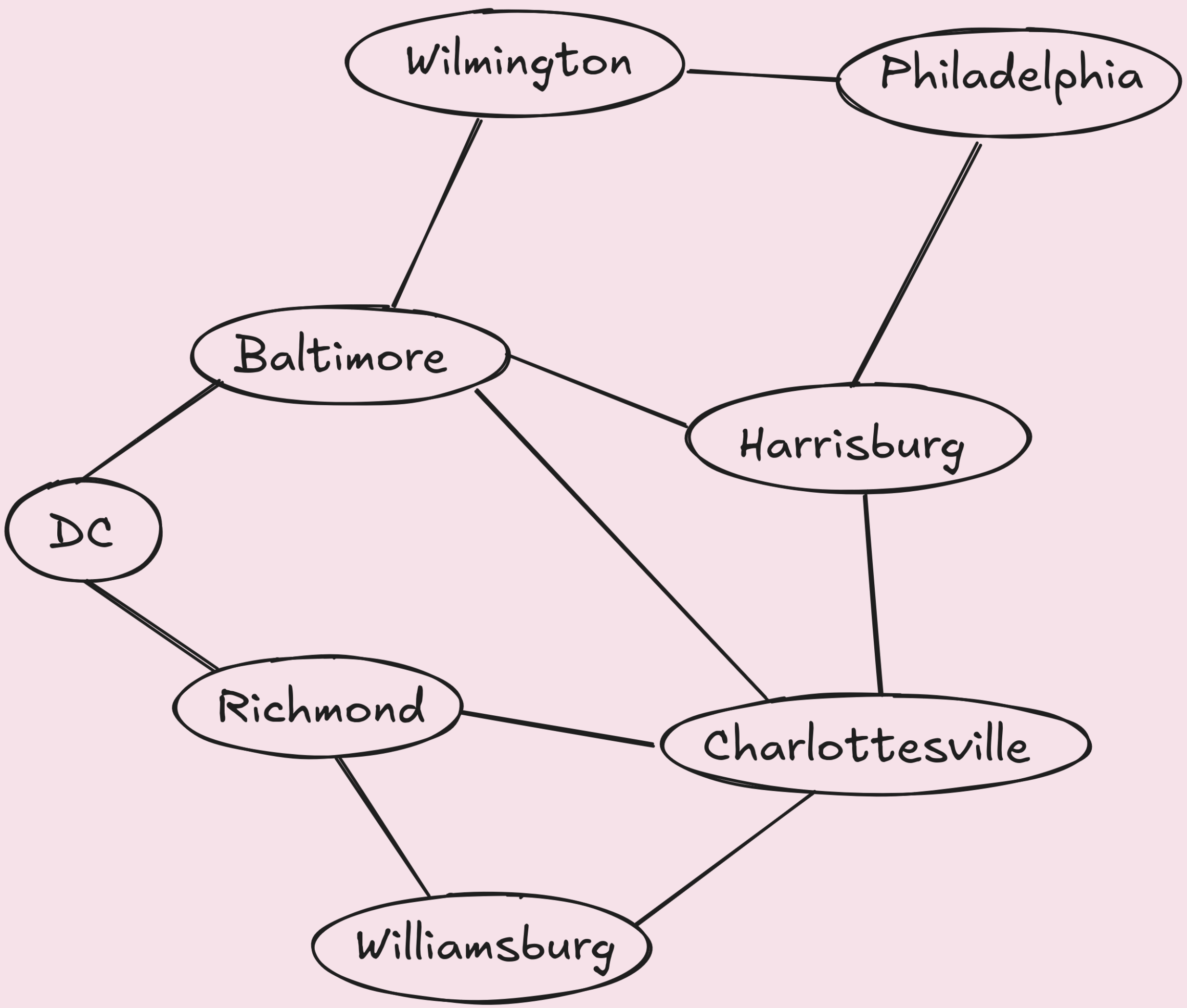
How to Choose?

- Think about when the searches “fail”
- Think about complexity
- Do we need an optimal solution?
  - Are we looking for “any” solution

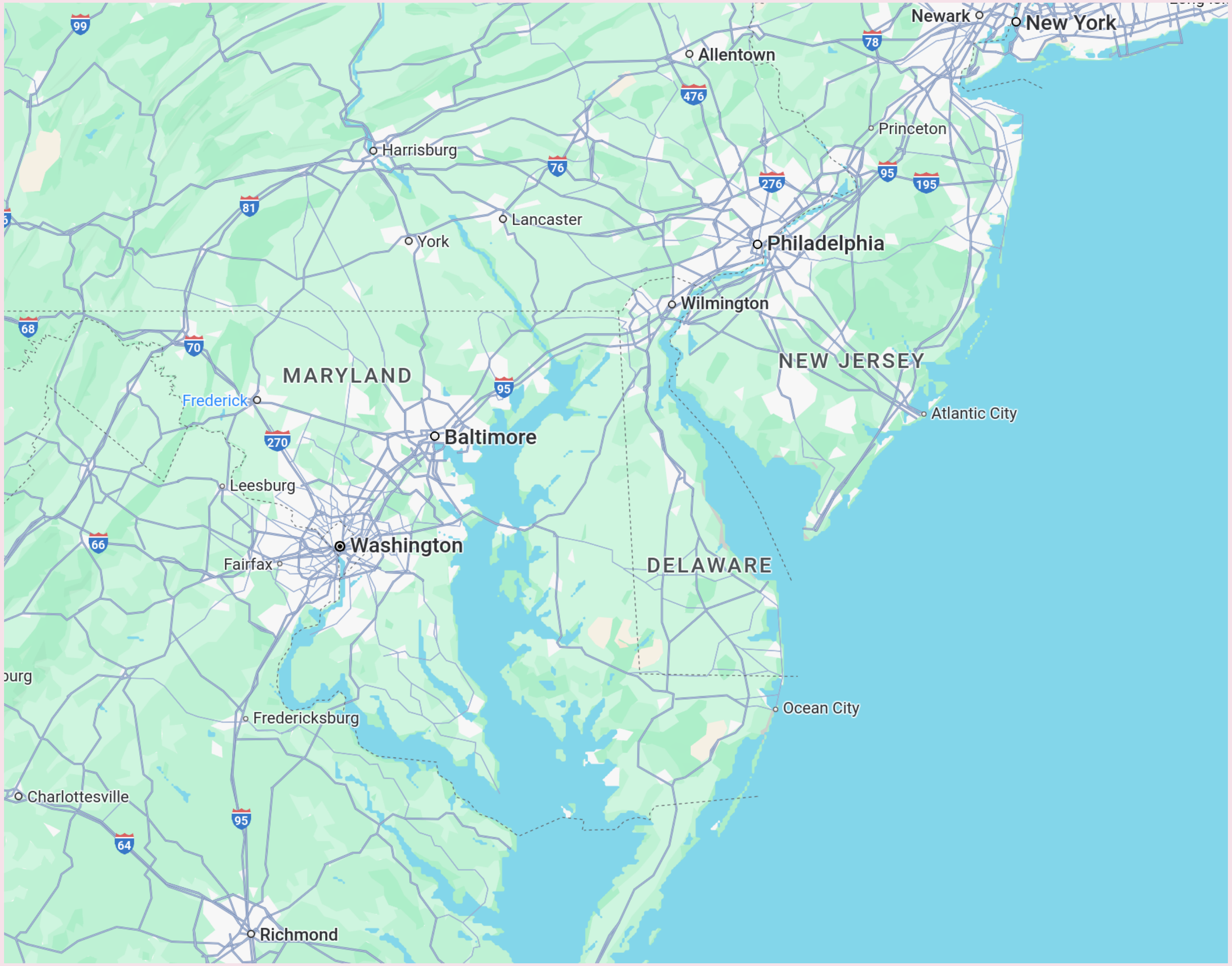
Informed Search

It Is Possible To Know Things

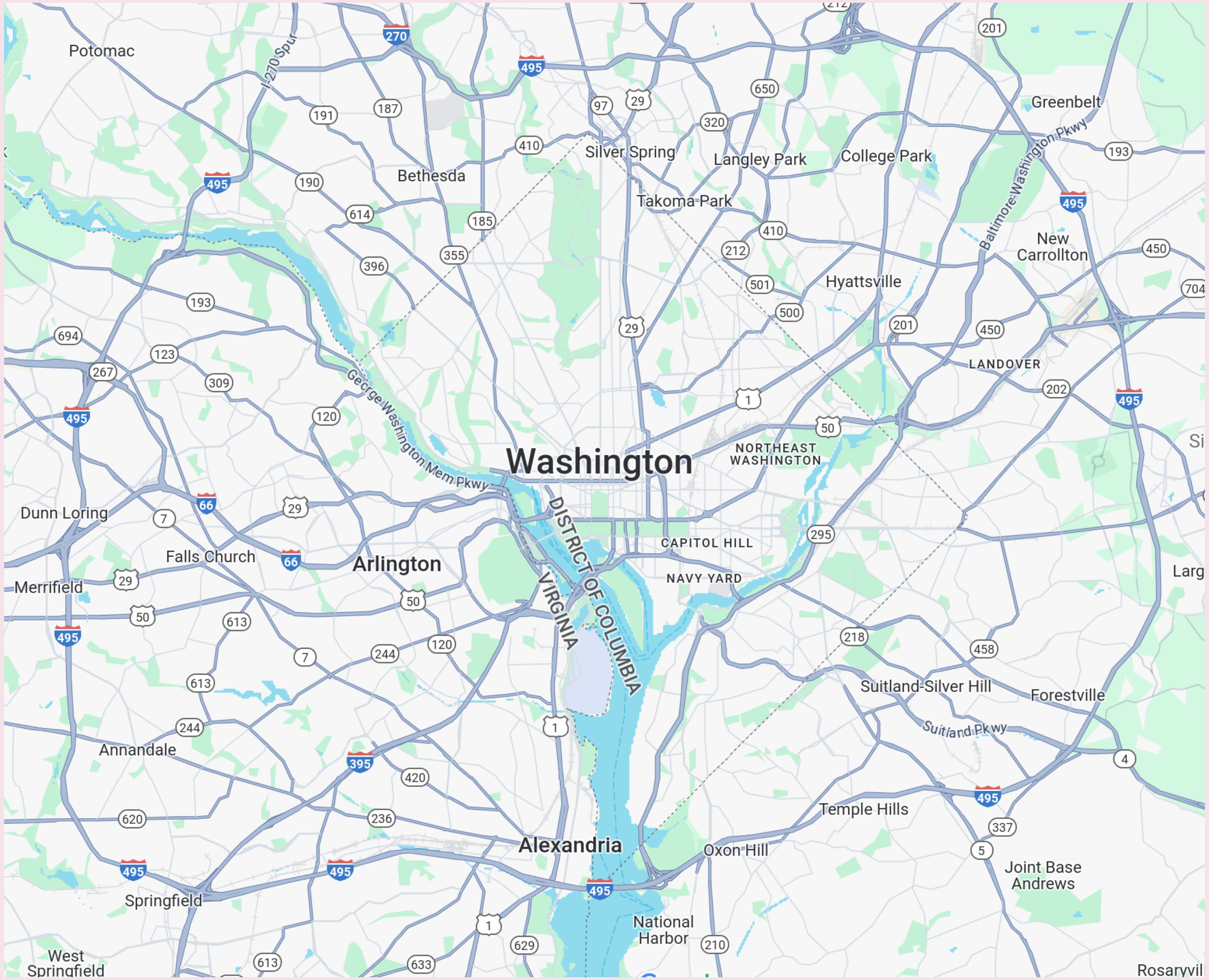
It Is Possible To Know Things



Mid-Atlantic



DC Metro Area



Heuristics

heuristic - *adj* - Serving to discover or find out.[1]

- We know things about the problem
- These things are external to the graph/tree structure
  - We could model the problem differently
  - We can use the information directly

1. Webster's, 1913

Best-First Search (reprise)

Algorithm Best-First Search

```
function BEST-FIRST-SEARCH(problem, f)
    node ← NODE(STATE=problem.INITIAL)
    frontier ← priority queue ordered by f
    frontier.ADD(node)
    reached ← lookup table
    reached[node] ← problem.INITIAL
    while not IS-EMPTY(frontier) do
        node ← POP(frontier)
        if problem.IS-GOAL(node.STATE) then
            return node
        for each child in EXPAND(problem,node) do
            s ← child.STATE
            if not s ∈ reached or child.PATH-COST < reached[s].PATH-COST then
                reached[s] ← child
                frontier.ADD(child)
    return failure

function EXPAND(problem, node)
    s ← node.STATE
    for each action in problem.ACTIONS(s) do
        s′ ← problem.RESULT(s, action)
        cost ← node.PATH-COST +problem.ACTION-COST(s, action, s′)
        yield NODE(STATE= s′, PARENT=node, ACTION=action, PATH-COST=cost)
```

Greedy Best-First Search

- Heuristic $h(n)$
  - n is the search-tree node
  - $h(n)$ estimates cost from n to goal
- Best-first search: $f(n)$ orders priority queue
  - Use $f(n) = h(n)$
- Complete
- No optimality guarantee
  - (expected)

A* Search

- Include path-cost $g(n)$
  - $f(n) = g(n) + h(n)$

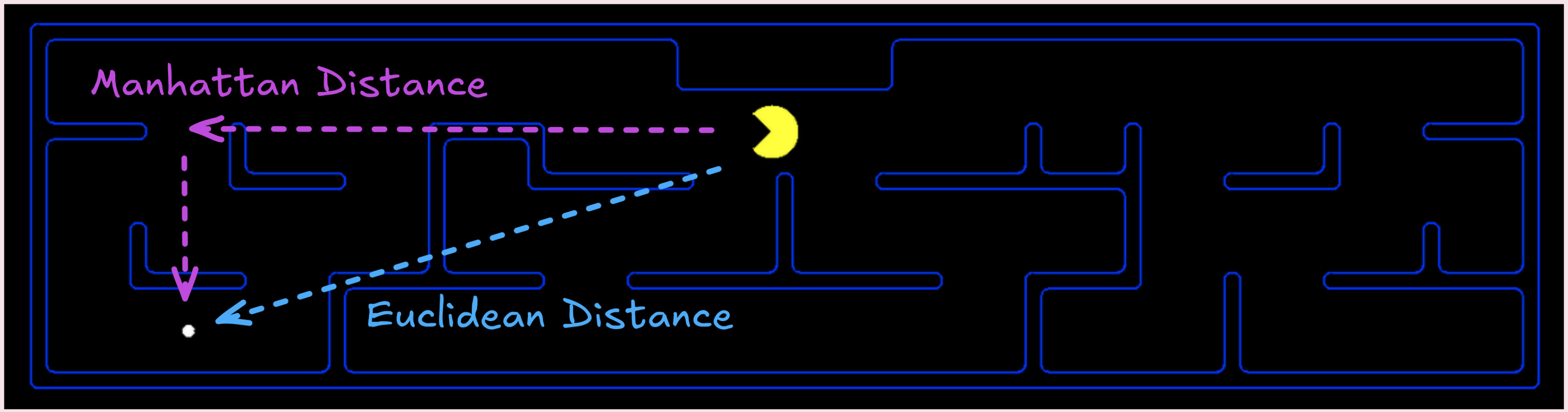
Algorithm A* Search

1: **function** A*-SEARCH($problem$)
2: **return** BEST-FIRST-SEARCH($problem$, $g(n) + h(n)$)

- Complete (always)
- Optimal (sometimes)
- Painful $O(b^m)$ time and space complexity

Choosing Heuristics

- Recall: $h(n)$ estimates cost from n to goal



- Admissibility
- Consistency

Choosing Heuristics

- Admissibility
  - *Never* overestimates cost from n to goal
  - Cost-optimal!
- Consistency
  - $h(n) \leq c(n, a, n') + h(n')$
  - n' successors of n
  - $c(n, a, n')$ cost from n to n' given action a

Consistency

- Consistent heuristics are admissible
  - Inverse not necessarily true
- Always reach each state on optimal path
- Implications for inconsistent heuristic?

Is Optimality Desirable?

Is Optimality Desirable?

- Yes

Is Optimality Desirable?

- Yes, but it isn't always *feasible*
  - A* search still exponentially complex in solution length
  - Optimality is never guaranteed "inexpensively"
- We need strategies for "good enough" solutions

Satisficing

satisfy - *verb* - To give satisfaction; to afford gratification; to leave nothing to be desired.[1]

suffice - *verb* - To be enough, or sufficient; to meet the need (of anything)[2]

1. Webster's, 1913
2. Webster's, 1913

Weighted A* Search

- Greedy: $f(n) = h(n)$
- A*: $f(n) = h(n) + g(n)$
- Uniform-Cost Search: $f(n) = g(n)$

...

- Weighted A* Search: $f(n) = W \cdot h(n) + g(n)$
  - Weight $W > 1$

Reducing Complexity

- Frontier Management
- Elimination of *reached* collection
    - Reference counts
    - How else?
- Other searches

Iterative-Deepening A* Search

“IDA*” Search

- Similar to Iterative Deepening with Depth-First Search
  - DFS uses depth cutoff
  - IDA* uses $h(n) + g(n)$ cutoff *with DFS*
  - Once cutoff breached, new cutoff:
    - Typically next-largest $h(n) + g(n)$
  - $O(b^m)$ time complexity 😔
  - $O(d)$ space complexity[1] 😌

1. This is slightly complicated based on heuristic branching factor b_h.

Beam Search

Best-First Search:

- Frontier is all expanded nodes

Beam Search:

- k "best" nodes are kept on frontier
  - Others discarded
- Alt: all nodes within δ of best node
- Not Optimal
- Not Complete

Recursive Best-First Search (RBFS)

- No *reached* table is kept
- Second-best node $f(n)$ retained
  - Search from each node cannot exceed this limit
  - If exceeded, recursion “backs up” to previous node
- Memory-efficient
  - Can “cycle” between branches

Recursive Best-First Search (RBFS)

Algorithm Recursive Best-First Search

```
function RECURSIVE-BEST-FIRST-SEARCH(problem)
    solution, f_value ← RFBS(problem,NODE(problem.INITIAL), ∞)
    return solution

function RBFS(problem, node, f_limit)
    if problem.IS-GOAL(node.STATE) then
        return node
    successors ← LIST(EXPAND(node))
    if IS-EMPTY(successors) then
        return failure, ∞
    for each s in successors do
        s.f ← MAX(s.PATH-COST + h(s), node.f)
    while True do
        best ← node in successors with lowest f
        if best.f > f_limit then
            return failure, best.f
        alternative ← node in successors with second-lowest f
        result, best.f ← RBFS(problem, best,MIN(f_limit, alterative))
        if result ≠ failure then
            return result, best.f
```

Heuristic Characteristics

- What makes a "good" heuristic?
  - We know about admissability and consistency
  - What about performance?
- Effective branching factor
- Effective depth
- # of nodes expanded

Where Do Heuristics Come From?

- Intuition
  - "Just Be Really Smart"
- Relaxation
  - The problem is constrained
  - Remove the constraint
- Pre-computation
  - Sub problems
- Learning

References

- Stuart J. Russell and Peter Norvig. *Artificial Intelligence: A Modern Approach*. 4th Edition, 2020.
- Stanford CS231
- UC Berkeley CS188